## NCERT Exemplar Class 9 Maths Chapter 1 Number Systems

### Exercise 1. 1

**1.** Every rational number is
(a) a natural number
(b) an integer
(c) a real number
(d) a whole number
**Solution:**
(c) Since, real numbers are the combination of rational and irrational numbers.
Hence, every rational number is a real number.

**2**.Between two rational numbers
(a) there is no rational number
(b) there is exactly one rational number
(c) there are infinitely many rational numbers
(d) there are only rational numbers and no irrational numbers
**Solution:**
(c) Between two rational numbers, there are infinitely many rational numbers.

**3.** Decimal representation of a rational number cannot be
(a) terminating
(b) non-terminating
(c) non-terminating repeating
(d) non-terminating non-repeating
**Solution:**
(d) Decimal representation of a rational number cannot be non-terminating non-repeating because the decimal expansion of rational number is either terminating or non-terminating recurring (repeating) or Non Terminating Repeating Numbers.

**4.** The product of any two irrational numbers is
(a) always an irrational number
(b) always a rational number
(c) always an integer
(d) sometimes rational, sometimes irrational
**Solution:**
(d) We know that, the product of any two irrational numbers is sometimes rational and sometimes irrational. It depends upon the given terms.

**5.** The decimal expansion of the number √2 is
(a) a finite decimal
(b) 1.41421
(c) non-terminating recurring
(d) non-terminating non-recurring
**Solution:**
The decimal expansion of the number √2 is non-terminating non-recurring. Because √2 is an irrational number.
Also, we know that an irrational number is non-terminating non-recurring.

**6.**Which of the following is irrational?
(a) $\sqrt{\frac{4}{9}}$ (b) $\frac{\sqrt{12}}{\sqrt{3}}$ (c) $\sqrt{7}$ (d) $\sqrt{81}$

**Solution:**
**(c)** √7is an irrational number, because √7 non-terminating non-recurring.

**Explanation:**

(A) √4/√9 = 2/3

(B) √12/√3 = 2√3/√3 = 2

(C) √7 = 2.64575131106

(D) √81 = 9

Here, (C) √7 = 2.64575131106 is a non-terminating decimal expansion.

Hence, (C) is the correct option.

**7.** Which of the following is irrational?
(a) 0.14 (b) $0.14\overline{16}$ (c) $0.\overline{1416}$ (d) 0.4014001400014
**Solution:** (D) 0.4014001400014…

**Explanation:**

A number is irrational if and only if its decimal representation is non-terminating and non-recurring.

(A) is a terminating decimal and, therefore, cannot be an irrational number.

(B) is a non-terminating and recurring decimal and, therefore, cannot be irrational.

(C) is a non-terminating and recurring decimal and, therefore, cannot be irrational.

(D) is a non-terminating and non-recurring decimal and therefore is an irrational number.

**8.** A rational number between √2 and √3 is

A rational number between $\sqrt{2}$ and $\sqrt{3}$ is

(a) $\frac{\sqrt{2}+\sqrt{3}}{2}$ (b) $\frac{\sqrt{2}\cdot\sqrt{3}}{2}$ (c) 1.5 (d) 1.8

**Solution:** (c) 1.5
**Explanation:**

√2 =1.4142135…. and √3 =1.732050807….

(A) (√2+√3)/2 = 1.57313218497… is a non-terminating and non-recurring decimal and, therefore, is an irrational number.

(B) (√2. √3)/2 = 1.22474487139… is a non-terminating and non-recurring decimal and, therefore, is an irrational number.

(C) 1.5 is a terminating decimal and, therefore, is a rational number.

(D) 1.8 is a terminating decimal and, therefore, is a rational number.

Here both 1.5 and 1.8 are rational numbers. But, 1.8 does not lie in between √2 =1.4142135…. and √3 =1.732050807…. Whereas 1.5 lies in between √2 =1.4142135…. and √3 =1.732050807….

**9.** The value of 1.999… in the form of p/q, where p and q are integers and q≠0, is
(a) 1910
(b) 19991000
(c) 2
(d) 19
**Solution: (c) 2**
**Explanation:**

(A) 19/10 = 1.9

(B) 1999/1000= 1.999

(C) 2

(D) 1/9 = 0.111….

Let x = 1.9999….. — (1)

Multiply equation (1) with 10

10x = 19.9999….. — (2)

Subtract equation (1) from equation(2),

We get,

9x = 18

x = 18 / 9

x = 2

Therefore,

x = 1.9999… = 2

Hence, (C) is the correct option.

**10.** 2√3 + √3 is equal to

(A) 2√6

(B) 6

(C) 3√3

(D) 4√6

**Solution:** (C) 3√3

Explanation:

2√3 + √3

Taking √3 common,

We get,

√3(2+1) = √3(3) = 3√3

Hence, (C) is the correct option.

**11.** √10 × √15 is equal to
(a) 6√5
(b) 5√6
(c) √25
(d) 10√5
**Solution:**
(b) 5√6
we have √10 × √15 = √2.√5 × √3.√5
= (√2.√3) × (√5.√5) = 5√6

**12.** The number obtained on rationalising the denominator of 1/ √7−2 is

(A) $\frac{\sqrt{7}+2}{3}$ (B) $\frac{\sqrt{7}-2}{3}$

(C) $\frac{\sqrt{7}+2}{5}$ (D) $\frac{\sqrt{7}+2}{45}$

**Solution:** (A)

: We have, $\frac{1}{\sqrt{7}-2}=\frac{1}{\sqrt{7}-2}\times\frac{\sqrt{7}+2}{\sqrt{7}+2}$

[By rationalising the denominator]

$=\frac{\sqrt{7}+2}{(\sqrt{7})^2-(2)^2}=\frac{\sqrt{7}+2}{7-4}=\frac{\sqrt{7}+2}{3}$

**13.**

$\frac{1}{\sqrt{9}-\sqrt{8}}$ is equal to

(A) $\frac{1}{2}(3-2\sqrt{2})$ (B) $\frac{1}{3+2\sqrt{2}}$

(C) $3-2\sqrt{2}$ (D) $3+2\sqrt{2}$

**Solution:**

**(D)** : We have, $\frac{1}{\sqrt{9}-\sqrt{8}}=\frac{1}{3-2\sqrt{2}}$

$=\frac{1}{3-2\sqrt{2}}\times\frac{3+2\sqrt{2}}{3+2\sqrt{2}}$

[By rationalising the denominator]

$=\frac{3+2\sqrt{2}}{9-(2\sqrt{2})^2}=\frac{3+2\sqrt{2}}{9-8}=3+2\sqrt{2}$

14.

After rationalising the denominator of $\frac{7}{3\sqrt{3}-2\sqrt{2}}$, we get the denominator as

(A) 13 (B) 19

(C) 5 (D) 35

**Solution:**

**(B)** : We have, $\frac{7}{3\sqrt{3}-2\sqrt{2}}$

$= \frac{7}{3\sqrt{3}-2\sqrt{2}} \times \frac{3\sqrt{3}+2\sqrt{2}}{3\sqrt{3}+2\sqrt{2}}$

[By rationalising the denominator]

$= \frac{7(3\sqrt{3}+2\sqrt{2})}{(3\sqrt{3})^2-(2\sqrt{2})^2} = \frac{7(3\sqrt{3}+2\sqrt{2})}{27-8}$

$= \frac{7(3\sqrt{3}+2\sqrt{2})}{19}$

Thus, after rationalising the denominator of $\frac{7}{3\sqrt{3}-2\sqrt{2}}$, we get 19 as denominator.

**15.**

**The value of $\frac{\sqrt{32}+\sqrt{48}}{\sqrt{8}+\sqrt{12}}$ is equal to**

**Solution:**

**(B)** : We have, $\frac{\sqrt{32}+\sqrt{48}}{\sqrt{8}+\sqrt{12}}$

$= \frac{4\sqrt{2}+4\sqrt{3}}{2\sqrt{2}+2\sqrt{3}} = \frac{4(\sqrt{2}+\sqrt{3})}{2(\sqrt{2}+\sqrt{3})} = 2$

**16.**

**If $\sqrt{2} = 1.4142$, then $\sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1}}$ is equal to**

**(A) 2.4142** **(B) 5.8282**
**(C) 0.4142** **(D) 0.1718**

**Solution:**

**(C)** : $\sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1}} = \sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1}}$

$= \sqrt{\frac{(\sqrt{2}-1)^2}{(\sqrt{2})^2-(1)^2}} = \sqrt{\frac{(\sqrt{2}-1)^2}{2-1}} = \sqrt{2}-1$

$= 1.4142 - 1 = 0.4142$

**17.**

**$\sqrt[4]{\sqrt[3]{2^2}}$ equals**

**Solution:**

**(C) :** $\sqrt[4]{\sqrt[3]{2^2}} = [\sqrt[3]{2^2}]^{1/4} = [(2^2)^{1/3}]^{1/4}$

$= [2^{2/3}]^{1/4} = 2^{\frac{2}{3}\cdot\frac{1}{4}} = 2^{\frac{1}{6}}$

**18.**

The product $\sqrt[3]{2}\cdot\sqrt[4]{2}\cdot\sqrt[12]{32}$ equals

(A) $\sqrt{2}$ (B) 2

(C) $\sqrt[12]{2}$ (D) $\sqrt[12]{32}$

**Solution:**

**(B) :** L.C.M. of 3, 4 and 12 = 12

$\therefore\ \sqrt[3]{2} = \sqrt[12]{2^4}, \sqrt[4]{2} = \sqrt[12]{2^3}$ and $\sqrt[12]{32} = \sqrt[12]{2^5}$

We have, $\sqrt[3]{2}\cdot\sqrt[4]{2}\cdot\sqrt[12]{32}$

$= \sqrt[12]{2^4}\cdot\sqrt[12]{2^3}\cdot\sqrt[12]{2^5}$

$= \sqrt[12]{2^4\cdot 2^3\cdot 2^5} = \sqrt[12]{2^{4+3+5}} = \sqrt[12]{2^{12}} = 2$

**19.**

Value of $\sqrt[4]{(81)^{-2}}$ is

(A) $\frac{1}{9}$ (B) $\frac{1}{3}$

(C) 9 (D) $\frac{1}{81}$

**Solution:**

**(A) :** We have, $\sqrt[4]{(81)^{-2}}$

$= \sqrt[4]{\frac{1}{(81)^2}} = \frac{1}{(81)^{2/4}} = \frac{1}{(81)^{1/2}}$

$= \frac{1}{(9^2)^{1/2}} = \frac{1}{9}$

**20.**

Value of $(256)^{0.16}\times(256)^{0.09}$ is

**Solution:**

(A) : We have, $(256)^{0.16} \times (256)^{0.09}$

$$= (256)^{\frac{16}{100}} \times (256)^{\frac{9}{100}} = (256)^{\frac{16}{100}+\frac{9}{100}}$$

$$= (256)^{\frac{25}{100}} = (256)^{\frac{1}{4}} = (4^4)^{\frac{1}{4}} = 4$$

**Q21.** Which of the following is equal to x?

(A) $x^{\frac{12}{7}} - x^{\frac{5}{7}}$ (B) $\sqrt[12]{(x^4)^{\frac{1}{3}}}$

(C) $(\sqrt{x^3})^{\frac{2}{3}}$ (D) $x^{\frac{12}{7}} \times x^{\frac{7}{12}}$

**Solution:**

(C) : (A) $x^{\frac{12}{7}} - x^{\frac{5}{7}} = x^{\frac{5}{7}+1} - x^{\frac{5}{7}}$

$$= x^{\frac{5}{7}} \cdot x - x^{\frac{5}{7}} = x^{\frac{5}{7}}(x-1) \neq x$$

(B) $\sqrt[12]{(x^4)^{\frac{1}{3}}} = \left((x^4)^{\frac{1}{3}}\right)^{\frac{1}{12}}$

$$= x^{4 \times \frac{1}{3} \times \frac{1}{12}} = x^{\frac{1}{9}} \neq x$$

(C) $(\sqrt{x^3})^{\frac{2}{3}} = (x^{\frac{3}{2}})^{\frac{2}{3}} = x$

(D) $x^{\frac{12}{7}} \times x^{\frac{7}{12}} = x^{\frac{12}{7}+\frac{7}{12}} = x^{\frac{193}{84}} \neq x$

## Exercise 1.2: Short Answer Type Questions

**Q1**. Let x and y be rational and irrational numbers, respectively. Is x+y necessarily an irrational number? Give an example in support of your answer.

**Solution:**

Yes, (x + y) is necessarily an irrational number.

For example, let x = 2 and y = √3

Then, x + y = 2 +√3

Suppose x + y = 2+ √3 be a rational number.

Let us consider a = 2 + √3, which is rational.

On squaring both sides, we get

$a^2 = (2+\sqrt{3})^2$

$\Rightarrow a^2 = 2^2 + (\sqrt{3})^2 + 2(2)(\sqrt{3})$

$\Rightarrow a^2 = 4 + 3 + 4\sqrt{3} \Rightarrow \frac{a^2-7}{4} = \sqrt{3}$

Since, $a$ is rational $\Rightarrow \frac{a^2-7}{4}$ is rational. So,

√3 is also a rational number.

The above contradicts the fact that √3 is an irrational number. Thus, our assumption was wrong. Hence, x + y is an irrational number.

**Q2.**Let x be rational and y be irrational. Is xy necessarily irrational? Justify your answer by an example.

**Solution:**

No, xy is necessarily irrational only when x ≠0.

Let x be a non-zero rational and y be an irrational. Then, we have to show that xy be an irrational. If possible, let xy be a rational number. Since quotient of two non-zero rational number is a rational number.

So,(xy/x) is a rational number

=> y is a rational number.

But, this contradicts the fact that y is an irrational number. Thus, our supposition is wrong. Hence, xy is an irrational number. But, when x = 0, then xy = 0, a rational number.

**Q3**.State whether the following statements are true or false? Justify your answer.

(i) 2√3 is a rational number.

(ii) There are infinitely many integers between any two integers.

(iii) Number of rational numbers between 15 and 18 is finite.

(iv)There are numbers which cannot be written in the from p/q, q≠0 p,q both are integers.

(v) The square of an irrational number is always rational.

**(vi) $\frac{\sqrt{12}}{\sqrt{3}}$ is not a rational number as $\sqrt{12}$ and $\sqrt{3}$ are not integers.**

**(vii) $\frac{\sqrt{15}}{\sqrt{3}}$ is written in the form $\frac{p}{q}$, $q \neq 0$ and so it is a rational number.**

**Solution:**

(i) False, here √2 is an irrational number and 3 is a rational number, we know that when we divide irrational number by non-zero rational number it will always give an irrational number.

(ii) False, because between two consecutive integers (like 1 and 2), there does not exist any other integer.

(iii) False, because between any two rational numbers there exist infinitely many rational numbers.

(iv) True, because there are infinitely many numbers which cannot be written in the form p/q, q ≠0. p,q both are integers and these numbers are called irrational numbers.

(v) Square of an irrational number is not always a rational number. For example let us consider an irrational number: 1+√2

So, $(1+\sqrt{2})^2 = 12 + \sqrt{2}^2 + 2 \times 1 \times \sqrt{2} = 1 + 2 + 2\sqrt{2} = 3 + 2\sqrt{2}$ which is an irrational number.

Thus, the given statement is False.

(vi) **False**

We have, $\frac{\sqrt{12}}{\sqrt{3}} = \frac{\sqrt{4} \times \sqrt{3}}{\sqrt{3}}$

= 2, which is a rational number.

(vii) **False**

We have, $\frac{\sqrt{15}}{\sqrt{3}} = \frac{\sqrt{5} \times \sqrt{3}}{\sqrt{3}} = \sqrt{5}$,

which is an irrational number.

**Q4.** Classify the following numbers as rational or irrational with justification

(i) $\sqrt{196}$ (ii) $3\sqrt{18}$

(iii) $\sqrt{\frac{9}{27}}$ (iv) $\frac{\sqrt{28}}{\sqrt{343}}$

(v) $-\sqrt{0.4}$ (vi) $\frac{\sqrt{12}}{\sqrt{75}}$

(vii) 0.5918 (viii) $(1+\sqrt{5})-(4+\sqrt{5})$

(ix) 10.124124... (x) 1.010010001...

**Solution:**

(i) We have, $\sqrt{196}=\sqrt{14\times14}=14$, which is a rational number.

(ii) We have, $3\sqrt{18}=3\sqrt{3\times3\times2}$
$=3\times3\sqrt{2}=9\sqrt{2}$, which is an irrational number.

(iii) We have, $\sqrt{\frac{9}{27}}=\sqrt{\frac{3\times3}{9\times3}}=\frac{1}{\sqrt{3}}$,
which is an irrational number, because $\sqrt{3}$ is an irrational number.

(iv) We have,
$\frac{\sqrt{28}}{\sqrt{343}}=\frac{\sqrt{2\times2\times7}}{\sqrt{7\times7\times7}}=\frac{2}{7}$,
which is a rational number.

(v) We have, $-\sqrt{0.4}=-\sqrt{\frac{4}{10}}=-\sqrt{\frac{2\times2}{5\times2}}$
$=-\sqrt{\frac{2}{5}}=-\frac{\sqrt{2}}{\sqrt{5}}$ which is an irrational number because $\sqrt{2}$ and $\sqrt{5}$ both are irrational numbers.

(vi) We have, $\frac{\sqrt{12}}{\sqrt{75}}=\frac{\sqrt{4\times3}}{\sqrt{25\times3}}=\frac{\sqrt{4}\sqrt{3}}{\sqrt{25}\sqrt{3}}=\frac{2}{5}$,
which is a rational number.

(vii) We have, 0.5918, whose decimal expansion is terminating and it can be written in the form of p/q, where q≠0, p and q are integers. Thus, 0.5918 is a rational number.

(viii) We have, $(1+\sqrt{5})-(4+\sqrt{5})$
$=1-4+\sqrt{5}-\sqrt{5}=-3$,
which is a rational number.

(ix) We have, 10.124124…, whose decimal expansion is non-terminating but recurring. Thus, 10.124124 is a rational number.

(x) We have 1.010010001…, whose decimal expansion is non-terminating non-recurring. Thus, 1.010010001… is an irrational number.

## Exercise 1.3: Short Answer Type Questions

**Q1.** Find which of the variables x, y, z and u represent rational numbers and which irrational numbers.

(i) $x^2 = 5$

(ii) $y^2 = 9$

(iii) $z^2 = 0.04$

(iv) $u^2 = 17/4$

**Solution:**

(i) We have, $x^2 = 5$

On taking square root on both sides, we get $x = \pm\sqrt{5}$, which is an irrational number.

(ii) We have, $y^2 = 9$

On taking square root on both sides, we get $y = \pm\sqrt{9} = \pm 3$, which is a rational number.

(iii) We have, $z^2 = .04$ or $z^2 = 0.04$

On taking square root on both sides, we get

$z = \pm\sqrt{0.04} = \pm\sqrt{\frac{4}{100}} = \pm\frac{2}{10}$, which is a rational number.

(iv) We have, $u^2 = \frac{17}{4}$

On taking square root on both sides, we get

$u = \pm\sqrt{\frac{17}{4}} = \pm\frac{\sqrt{17}}{2}$, which is an irrational number.

**Q2.** Find three rational numbers between

(i) -1 and -2

(ii) 0.1 and 0.11

(iii) 5/7 and 6/7

(iv) 1/4 and 1/5

**Solution:**

(i) Let $x = -1$ and $y = -2$

As we know, a rational number between $x$ and $y$ can be find out as $\frac{x+y}{2}$.

$\therefore$ A rational number between $-1$ and $-2$ is $\frac{-1-2}{2} = -\frac{3}{2}$ and a rational number between $-1$ and $-\frac{3}{2} = \frac{-1-\frac{3}{2}}{2} = \frac{-2-3}{4} = -\frac{5}{4}$

Similarly, a rational number between $-\frac{3}{2}$ and $-2$ is $-\frac{7}{4}$.

Thus, the three rational numbers between $-1$ and $-2$ are $-\frac{3}{2}, -\frac{5}{4}$ and $-\frac{7}{4}$

(ii) Let x = 0.1 and y = 0.11

As we can write, x = 0.1 = 0.100 and y = 0.11 = 0.110

Thus, the three rational numbers between 0.100 and 0.110 are 0.101, 0.102, 0.103.

(iii) Let $x = \frac{5}{7}$ and $y = \frac{6}{7}$

We can write

$$x = \frac{5}{7} = \frac{5\times10}{7\times10} = \frac{50}{70}$$

and $y = \frac{6}{7} = \frac{6\times10}{7\times10} = \frac{60}{70}$

Thus, the three rational numbers between

$\frac{5}{7}$ and $\frac{6}{7}$ are $\frac{51}{70}, \frac{52}{70}$ and $\frac{53}{70}$

(iv) Let $x = \frac{1}{4}$ and $y = \frac{1}{5}$

As we know that a rational number between $x$ and $y$ is $\frac{x+y}{2}$

$\therefore$ A rational number between $\frac{1}{4}$ and $\frac{1}{5}$

$$= \frac{\frac{1}{4}+\frac{1}{5}}{2} = \frac{\frac{5+4}{20}}{2} = \frac{9}{2\times20} = \frac{9}{40}$$

Now a rational number between $\frac{1}{4}$ and $\frac{9}{40}$ is

$$= \frac{\frac{1}{4}+\frac{9}{40}}{2} = \frac{\frac{10+9}{40}}{2} = \frac{19}{2\times40} = \frac{19}{80}$$

Again, a rational number between $\frac{1}{5}$ and $\frac{9}{40}$ is

$$= \frac{\frac{1}{5}+\frac{9}{40}}{2} = \frac{\frac{8+9}{40}}{2} = \frac{17}{40\times2} = \frac{17}{80}$$

Thus, the three rational numbers between

$\frac{1}{4}$ and $\frac{1}{5}$ are $\frac{9}{40}, \frac{19}{80}$ and $\frac{17}{80}$.

(iv) 1/4 and 1/5

Here, according to the question,

LCM of 4 and 5 is 20.

Let us make the denominators common, 80.

(4 × 20) = 80 and (5 × 16) = 80

Hence,

1/4 can be written as (1 × 20)/(4 × 20) = 20/80

Similarly,

1/5 can be written as (1 × 16)/(5 × 16) = 16/80

**Q 3. Insert a rational number and an irrational number between the following**

(i) 2 and 3

(ii) 0 and 0.1

(iii) 1/3 and 1/2

(iv) -2/5 and -1/2

(v) 0.15 and 0.16

(vi) √2 and √3

(vii) 2.357 and 3.121
(viii) .0001 and .001
(ix) 3.623623 and 0.484848
(x) 3.375289 and 6.375738

**Solution:**

We know that, there are infinitely many rational and irrational values between any two numbers.

**(i)** A rational number between 2 and 3 is 2.1.
To find an irrational number between 2 and 3. Find a number which is non-terminating non-recurring lying between them.
Such number will be 2.040040004…………..

**(ii)** A rational number between 0 and 0.1 is 0.03.
An irrational number between 0 and 0.1 is 0.007000700007……….

**(iii)** LCM of 3 and 2 is 6.

1/3 = 0.33

1/3 can be written as (1 × 20)/(3 × 20) = 20/60

½ = 0.5

1/2 can be written as (1 × 30)/(2 × 30) = 30/60

So, the rational number between 1/3 and 1/2 = 25/60

And, irrational number between 1/3 and 1/2 = irrational number between 0.33 and 0.5 = 0.414114111…

**(iv)** LCM of 5 and 2 is 10.

-2/5 = -0.4

-2/5 can be written as (-2 × 2)/(5 × 2) = -4/10

1/2 = 0.5

1/2 can be written as (1 × 5)/(2 × 5) = 5/10

So, rational number between -2/5 and 1/2 = rational number between -4/10 and 5/10 = 1/10

And, irrational number between -2/5 and 1/2 = irrational number between -0.4 and 0.5 = 0.414114111…

**(v)** A rational number between 0.15 and 0.16 is 0.151. An irrational number between 0.15 and 0.16 is 0.1515515551…….

**(vi)** A rational number between √2 and √3 i.e.,, between 1.4142…… and 1.7320…… is 1.5. An irrational number between √2 and √3 is 1.585585558……….

**(vii)** A rational number between 2.357 and 3.121 is 3. An irrational number between 2.357 and 3.121 is 3.101101110……..

**(viii)** A rational number between 0.0001 and 0.001 is 0.00011. An irrational number between 0.0001 and 0.001 is 0.0001131331333………..

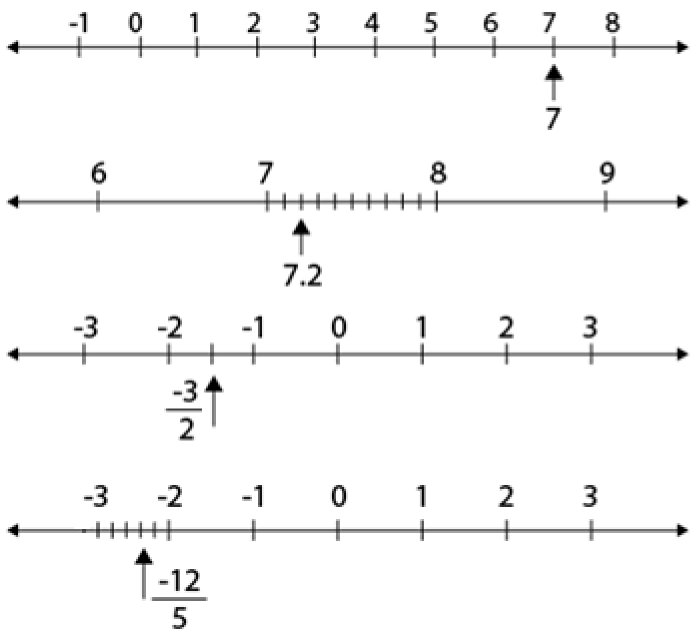
**(ix)** A rational number between 3.623623 and 0.484848 is 1. An irrational number between 3.623623 and 0.484848 is 1.909009000……….

**(x)** A rational number between 6.375289 and 6.375738 is 6.3753. An irrational number between 6.375289 and 6.375738 is 6.375414114111………

**Q4.** Represent the following numbers on the number line:

**7, 7.2, −3/2, −12/5**

**Solution:**



**5. Locate √5, √10 and √17 on the number line.**

**Solution:**

√5 on the number line:

5 can be written as the sum of the square of two natural numbers:

i.e., 5 =1+ 4 =12 + 22

On the number line,

Take OA = 2 units.

Perpendicular to OA, draw BA = 1 unit.

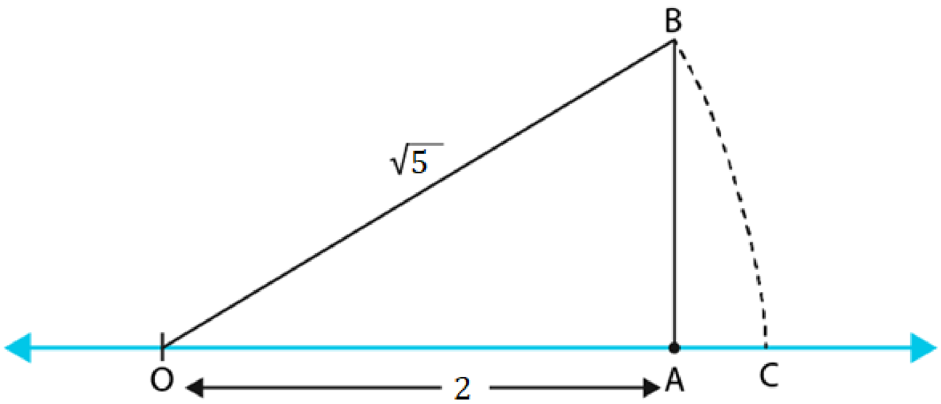
Join OB.

Using the Pythagoras theorem,

We have, OB= √5

Draw an arc with centre O and radius OB using a compass such that it intersects the number line at point C.

Then, we get, C corresponds to √5. Or we can say that OC = √5




√10 on the number line:

10 can be written as the sum of the square of two natural numbers:

i.e., 10 =1+ 9 =12 + 32

On the number line,

Take OA = 3 units.

Perpendicular to OA, draw BA = 1 unit.

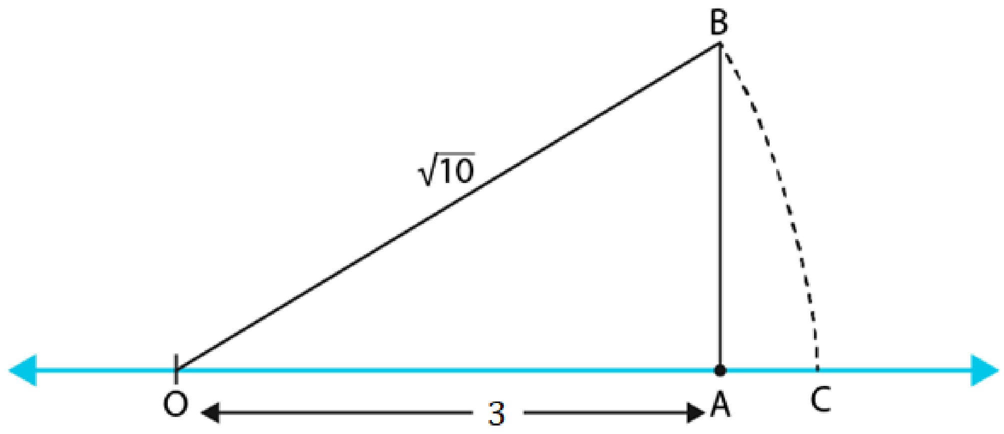
Join OB.

Using the Pythagoras theorem,

We have, OB= √10

Draw an arc with centre O and radius OB using a compass such that it intersects the number line at point C.

Then, point C corresponds to √10. Or we can say that OC = √10




√17 on the number line:

17 can be written as the sum of the square of two natural numbers:

i.e., 17 =1+ 16 =12 + 42

On the number line,

Take OA = 4 units.

Perpendicular to OA, draw BA = 1 unit.

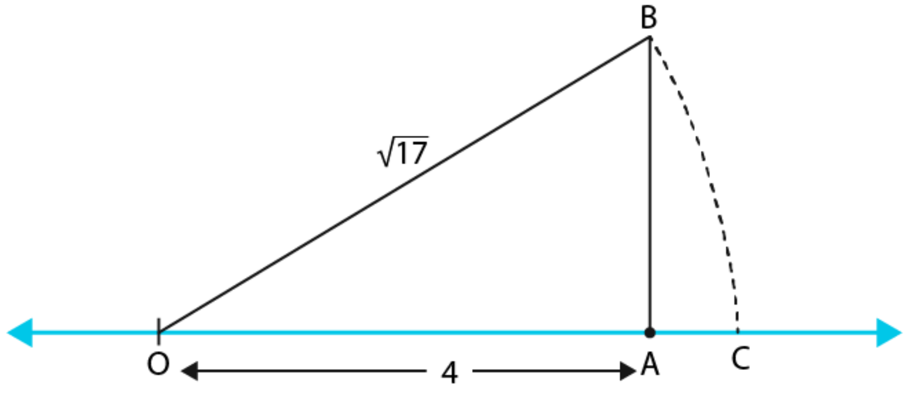
Join OB.

Using the Pythagoras theorem,

We have, OB= √17

Draw an arc with centre O and radius OB using a compass such that it intersects the number line at point C.

Then, point C corresponds to √17. Or, we can say that OC = √17




**6. Represent geometrically the following numbers on the number line:**

**(i) $\sqrt{4.5}$**

**(ii) $\sqrt{5.6}$**

**(iii) $\sqrt{8.1}$**

**(iv) $\sqrt{2.3}$**

**Solution:**

**(i)** $\sqrt{4.5}$

Draw a line segment such that AB = 4.5 units.

Mark C at a distance of 1 unit from B.

Mark O is the mid-point of AC.

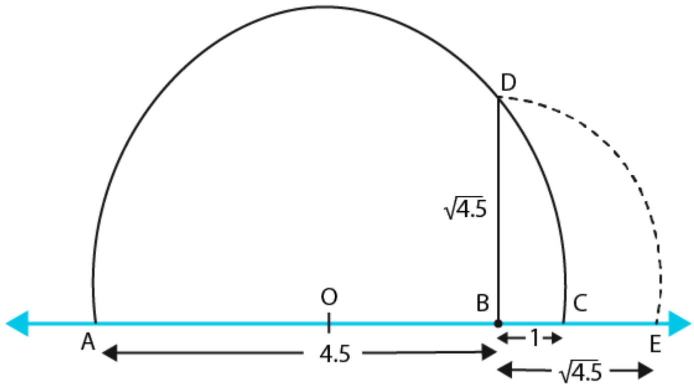
Draw a semicircle with centre O and radius OC.

Draw a line perpendicular to AC, passing through B and intersecting the semicircle at D.

Now, BD = $\sqrt{4.5}$.

Draw an arc with centre B and radius BD, meeting AC produced at E.

Then, BE = BD = $\sqrt{4.5}$ units.




**(ii)** √5.6

Draw a line segment such that AB = 5.6 units.

Mark C at a distance of 1 unit from B.

Mark O is the mid-point of AC.

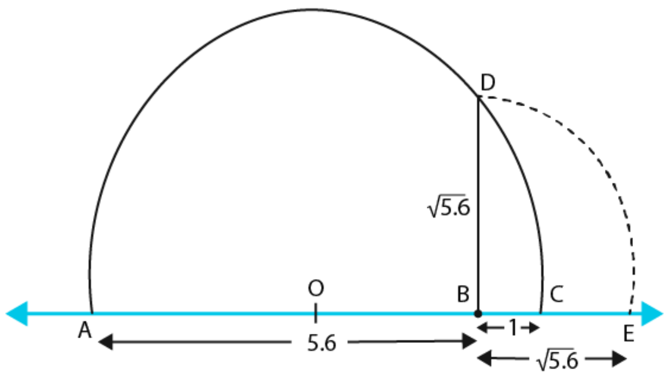
Draw a semicircle with centre O and radius OC.

Draw a line perpendicular to AC, passing through B and intersecting the semicircle at D.

Now, BD = √5.6

Draw an arc with centre B and radius BD, meeting AC produced at E.

Then BE = BD = √5.6 units.




**(iii)** √8.1

Draw a line segment such that AB = 8.1 units.

Mark C at a distance of 1 unit from B.

Mark O, is the mid-point of AC.

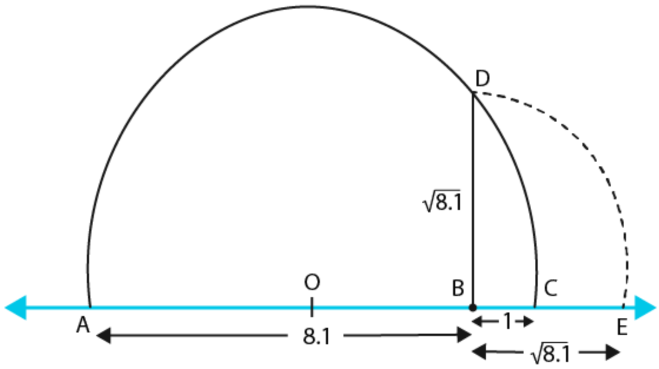
Draw a semicircle with centre O and radius OC.

Draw a line perpendicular to AC, passing through B and intersecting the semicircle at D.

Now, BD = $\sqrt{8.1}$.

Draw an arc with centre B and radius BD, meeting AC produced at E.

Then BE = BD = $\sqrt{8.1}$ units.




**(iv)** $\sqrt{2.3}$

Draw a line segment such that AB = 2.3 units.

Mark C at a distance of 1 unit from B.

Mark O is the mid-point of AC.

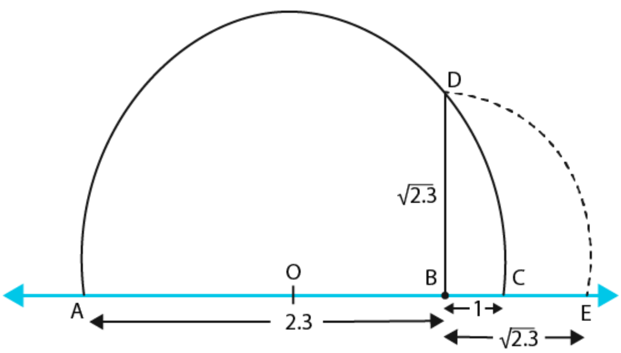
Draw a semicircle with centre O and radius OC.

Draw a line perpendicular to AC, passing through B and intersecting the semicircle at D.

Now, BD = $\sqrt{2.3}$.

Draw an arc with centre B and radius BD, meeting AC produced at E.

Then BE = BD = $\sqrt{2.3}$ units.



**Q7. Express the following in the form p/q, where p and q are integers and q ≠ 0 :**

**(i) 0.2**

**(ii) 0.888…**

**(iii)** $5.\overline{2}$

**(iv)** $0.\overline{001}$

**(v) 0.2555…**

**(vi)** $0.1\overline{34}$

**(vii) .00323232…**

**(viii) .404040…**

**Solution:**

**(i) 0.2**

We know that,

0/2 can be written as,

0.2 = 2/10 = 1/5

**(ii)** 0.888…

Assume that $x$ = 0.888 …

⇒ $x$ = 0.8 ……………. Eq.(1)

Multiply L.H.S and R.H.S by 10,

We get

10 $x$ = 8.8 ……………. Eq.(2)

Subtracting equation (1) from (2),

We get

10 $x$ − $x$ = 8.8 − 0.8

⇒ 9$x$ = 8

⇒ $x$ = 8/9

**(iii)**

$5.\overline{2}$

Assume that $x$ = 5.2 ……………. Eq.(1)

Multiply L.H.S and R.H.S by 10,

We get

10 $x$ = 52.2 …………… Eq. (2)

Subtracting equation (1) from (2),

We get

10 $x$ − $x$ = 52.2 − 5.2

⇒ 9$x$ = 47

⇒ $x$ = 47/9

**(iv)**

$0.\overline{001}$

Assume that $x$ = 0.001 ……………. Eq. (1)

Multiply L.H.S and R.H.S by 1000,

We get

1000 $x$ = 1.001 …………… Eq. (2)

Subtracting equation (1) from (2),

We get

1000$x$ − $x$ = 1.001 − 0.001

⇒ 999$x$ = 1

⇒ $x$ = 1/999

**(v)** 0.2555…

Assume that $x$ = 0.2555 …

⇒ x = 0.25 ……………. Eq. (1)

Multiply L.H.S and R.H.S by 10,

We get

10 x = 2.5 ……………. Eq. (2)

Multiply L.H.S and R.H.S by 100,

We get

100 x = 25.5 …………. Eq. (3)

Subtracting equation (2) from (3),

We get

100 x-10x = 25.5 – 2.5

⇒ 90$x$ = 23

⇒ $x$ = 23/90

**(vi)**

$0.1\overline{34}$

Let $x$ = 0.134 …………….. Eq. (1)

Multiply L.H.S and R.H.S by 10,

We get

10 $x$ = 1.34 ………………. Eq. (2)

Multiply L.H.S and R.H.S by 1000,

We get

1000 $x$ = 134.34 …………. Eq. (3)

Subtracting equation (2) from (3),

We get

1000 $x$ − 10$x$ = 134.34 − 1.34

⇒ 990$x$ = 133

⇒ $x$ = 133/990

(vii) .00323232…

Let $x$ = 0.00323232 …

⇒ x = 0.0032 ……………. Eq. (1)

Multiply L.H.S and R.H.S by 100,

We get,

100x = 0.32 ……………. Eq. (2)

Multiply L.H.S and R.H.S by 10000,

We get

10000 x = 32.32 …………. Eq. (3)

Subtracting equation (2) from (3),

We get

10000 x-100x = 32.32 – 0.32

⇒ 9900$x$ = 32

⇒ $x$ = 32/9900 = 8/2475

**(viii) .**404040…

Let $x$ = 0.404040 …

⇒ $x$ = 0. 40 ………………. (1)

Multiply L.H.S and R.H.S by 100,

We get

100 $x$ = 40.40 ………….. (2)

Subtracting equation (1) from (2),

We get

100 $x - x$ = 40.40 − 0.40

⇒ 99$x$ = 40

⇒ $x$ = 40/99

**Q8.** Show that 0.142857142857… = 1/7

**Solution:**

Let $x = 0.142857142857...$ ...(1)

On multiplying both sides of (1) by 1000000, we get

$1000000\,x = 142857.142857...$ ...(2)

On subtracting (1) from (2), we get

$1000000x - x = (142857.142857...) - (0.142857...)$

$\Rightarrow \quad 999999x = 142857$

Thus, $x = \frac{142857}{999999} = \frac{1}{7}$

**Q9.** Simplify the following

(i) $\sqrt{45}-3\sqrt{20}+4\sqrt{5}$ (ii) $\frac{\sqrt{24}}{8}+\frac{\sqrt{54}}{9}$

(iii) $\sqrt[4]{12}\times\sqrt[7]{6}$ (iv) $4\sqrt{28}\div3\sqrt{7}\div\sqrt[3]{7}$

(v) $3\sqrt{3}+2\sqrt{27}+\frac{7}{\sqrt{3}}$ (vi) $(\sqrt{3}-\sqrt{2})^2$

(vii) $\sqrt[4]{81}-8\sqrt[3]{216}+15\sqrt[5]{32}+\sqrt{225}$

(viii) $\frac{3}{\sqrt{8}}+\frac{1}{\sqrt{2}}$ (ix) $\frac{2\sqrt{3}}{3}-\frac{\sqrt{3}}{6}$

**Solution:**

(i) We have, $\sqrt{45}-3\sqrt{20}+4\sqrt{5}$

$=3\sqrt{5}-6\sqrt{5}+4\sqrt{5}=\sqrt{5}$

(ii) We have, $\frac{\sqrt{24}}{8}+\frac{\sqrt{54}}{9}$

$=\frac{2\sqrt{6}}{8}+\frac{3\sqrt{6}}{9}=\frac{\sqrt{6}}{4}+\frac{\sqrt{6}}{3}$

$=\frac{3\sqrt{6}+4\sqrt{6}}{12}=\frac{7\sqrt{6}}{12}$

(iii) We have, $\sqrt[4]{12}\times\sqrt[7]{6}=(12)^{1/4}\times(6)^{1/7}$

$=(2\times2\times3)^{1/4}\times(2\times3)^{1/7}$

$=2^{1/4}\cdot2^{1/4}\cdot3^{1/4}\cdot2^{1/7}\cdot3^{1/7}$

$=2^{\frac{1}{4}+\frac{1}{4}+\frac{1}{7}}\times3^{\frac{1}{4}+\frac{1}{7}}=2^{9/14}\times3^{11/28}$

$=\sqrt[14]{2^9}\cdot\sqrt[28]{3^{11}}=\sqrt[28]{2^{18}}\cdot\sqrt[28]{3^{11}}$

$=\sqrt[28]{2^{18}\times3^{11}}$

(iv) We have, $4\sqrt{28}\div3\sqrt{7}\div\sqrt[3]{7}$

$=\left(\frac{8\sqrt{7}}{3\sqrt{7}}\right)\div(7)^{\frac{1}{3}}=\frac{8}{3}\div7^{\frac{1}{3}}=\frac{8}{3\sqrt[3]{7}}$

(v) We have, $3\sqrt{3}+2\sqrt{27}+\frac{7}{\sqrt{3}}$

$=3\sqrt{3}+6\sqrt{3}+\frac{7\sqrt{3}}{3}=9\sqrt{3}+\frac{7\sqrt{3}}{3}$

$=\frac{27\sqrt{3}+7\sqrt{3}}{3}=\frac{34\sqrt{3}}{3}$

(vi) We have, $\left(\sqrt{3}-\sqrt{2}\right)^2$

$=\left(\sqrt{3}\right)^2+\left(\sqrt{2}\right)^2-2\sqrt{3}\times\sqrt{2}$

$=3+2-2\sqrt{3\times2}=5-2\sqrt{6}$

(vii) We have, $\sqrt[4]{81}-8\sqrt[3]{216}+15\sqrt[5]{32}+\sqrt{225}$

$=(3^4)^{\frac{1}{4}}-8\times(6^3)^{\frac{1}{3}}+15\times(2^5)^{\frac{1}{5}}+15$

$=3-48+30+15=0$

(viii) We have, $\frac{3}{\sqrt{8}}+\frac{1}{\sqrt{2}}=\frac{3}{2\sqrt{2}}+\frac{1}{\sqrt{2}}$

$=\frac{3+2}{2\sqrt{2}}=\frac{5}{2\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$

[By rationalising the denominator]

$=\frac{5\sqrt{2}}{4}$

(ix) We have, $\frac{2\sqrt{3}}{3}-\frac{\sqrt{3}}{6}$

$=\frac{4\sqrt{3}-\sqrt{3}}{6}=\frac{3\sqrt{3}}{6}=\frac{\sqrt{3}}{2}$

**Q10**. Rationalise the denominator of the following

**(i)** $\frac{2}{3\sqrt{3}}$ **(ii)** $\frac{\sqrt{40}}{\sqrt{3}}$

**(iii)** $\frac{3+\sqrt{2}}{4\sqrt{2}}$ **(iv)** $\frac{16}{\sqrt{41}-5}$

**(v)** $\frac{2+\sqrt{3}}{2-\sqrt{3}}$ **(vi)** $\frac{\sqrt{6}}{\sqrt{2}+\sqrt{3}}$

**(vii)** $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ **(viii)** $\frac{3\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$

**(ix)** $\frac{4\sqrt{3}+5\sqrt{2}}{\sqrt{48}+\sqrt{18}}$

**Solution:**

(i) We have, $\frac{2}{3\sqrt{3}} = \frac{2}{3\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{2\sqrt{3}}{3\times 3} = \frac{2\sqrt{3}}{9}$

(ii) We have, $\frac{\sqrt{40}}{\sqrt{3}} = \frac{\sqrt{40}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{\sqrt{40\times 3}}{3} = \frac{2}{3}\sqrt{30}$

(iii) We have, $\frac{3+\sqrt{2}}{4\sqrt{2}} = \frac{3+\sqrt{2}}{4\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{3\sqrt{2}+2}{4\times 2} = \frac{3\sqrt{2}+2}{8}$

(iv) We have, $\frac{16}{\sqrt{41}-5} = \frac{16}{\sqrt{41}-5} \times \frac{\sqrt{41}+5}{\sqrt{41}+5}$

$= \frac{16(\sqrt{41}+5)}{(\sqrt{41})^2-(5)^2} = \frac{16(\sqrt{41}+5)}{41-25}$

$= \frac{16(\sqrt{41}+5)}{16} = \sqrt{41}+5$

(v) We have, $\frac{2+\sqrt{3}}{2-\sqrt{3}} = \frac{2+\sqrt{3}}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$

$= \frac{(2+\sqrt{3})^2}{(2)^2-(\sqrt{3})^2} = 4+3+4\sqrt{3} = 7+4\sqrt{3}$

(vi) We have, $\frac{\sqrt{6}}{\sqrt{2}+\sqrt{3}}$

$= \frac{\sqrt{6}}{\sqrt{2}+\sqrt{3}} \times \frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}-\sqrt{3}}$

$= \frac{\sqrt{6}(\sqrt{2}-\sqrt{3})}{(\sqrt{2})^2-(\sqrt{3})^2} = \frac{\sqrt{6}(\sqrt{2}-\sqrt{3})}{2-3}$

$= \frac{\sqrt{6}(\sqrt{2}-\sqrt{3})}{-1} = \sqrt{6}(\sqrt{3}-\sqrt{2})$

$= 3\sqrt{2}-2\sqrt{3}$

(vii) We have, $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

$$=\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}\times\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$$

$$=\frac{(\sqrt{3}+\sqrt{2})^2}{(\sqrt{3})^2-(\sqrt{2})^2}=3+2+2\sqrt{6}=5+2\sqrt{6}$$

(viii) We have, $\frac{3\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$

$$=\frac{3\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}\times\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$

$$=\frac{3\sqrt{5}(\sqrt{5}+\sqrt{3})+\sqrt{3}(\sqrt{5}+\sqrt{3})}{(\sqrt{5})^2-(\sqrt{3})^2}$$

$$=\frac{15+3\sqrt{15}+\sqrt{15}+3}{5-3}$$

$$=\frac{18+4\sqrt{15}}{2}=9+2\sqrt{15}$$

(ix) We have, $\frac{4\sqrt{3}+5\sqrt{2}}{\sqrt{48}+\sqrt{18}}=\frac{4\sqrt{3}+5\sqrt{2}}{4\sqrt{3}+3\sqrt{2}}$

$$=\frac{4\sqrt{3}+5\sqrt{2}}{4\sqrt{3}+3\sqrt{2}}\times\frac{4\sqrt{3}-3\sqrt{2}}{4\sqrt{3}-3\sqrt{2}}$$

$$=\frac{4\sqrt{3}(4\sqrt{3}-3\sqrt{2})+5\sqrt{2}(4\sqrt{3}-3\sqrt{2})}{(4\sqrt{3})^2-(3\sqrt{2})^2}$$

$$=\frac{48-12\sqrt{6}+20\sqrt{6}-30}{30}$$

$$=\frac{18+8\sqrt{6}}{30}=\frac{9+4\sqrt{6}}{15}$$

**Q11**.Find the values of a and b in each of the following

(i) $\frac{5+2\sqrt{3}}{7+4\sqrt{3}} = a - 6\sqrt{3}$

(ii) $\frac{3-\sqrt{5}}{3+2\sqrt{5}} = a\sqrt{5} - \frac{19}{11}$

(iii) $\frac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}} = 2 - b\sqrt{6}$

(iv) $\frac{7+\sqrt{5}}{7-\sqrt{5}} - \frac{7-\sqrt{5}}{7+\sqrt{5}} = a + \frac{7}{11}\sqrt{5}b$

**Solution:**

(i) We have, $\frac{5+2\sqrt{3}}{7+4\sqrt{3}} = a - 6\sqrt{3}$

By rationalising the denominator on L.H.S. of above equation, we get

$\frac{5+2\sqrt{3}}{7+4\sqrt{3}} \times \frac{7-4\sqrt{3}}{7-4\sqrt{3}} = a - 6\sqrt{3}$

$\Rightarrow \frac{5(7-4\sqrt{3})+2\sqrt{3}(7-4\sqrt{3})}{7^2-(4\sqrt{3})^2} = a - 6\sqrt{3}$

$\Rightarrow \frac{35-20\sqrt{3}+14\sqrt{3}-24}{49-48} = a - 6\sqrt{3}$

$\Rightarrow 11 - 6\sqrt{3} = a - 6\sqrt{3}$

Thus, $a = 11$

(ii) We have, $\frac{3-\sqrt{5}}{3+2\sqrt{5}} = a\sqrt{5} - \frac{19}{11}$

By rationalising the denominator on L.H.S. of above equation, we get

$\Rightarrow \frac{3-\sqrt{5}}{3+2\sqrt{5}} \times \frac{3-2\sqrt{5}}{3-2\sqrt{5}} = a\sqrt{5} - \frac{19}{11}$

$\Rightarrow \frac{3(3-2\sqrt{5})-\sqrt{5}(3-2\sqrt{5})}{(3)^2-(2\sqrt{5})^2} = a\sqrt{5} - \frac{19}{11}$

$\Rightarrow \frac{9-6\sqrt{5}-3\sqrt{5}+10}{9-20} = a\sqrt{5} - \frac{19}{11}$

$\Rightarrow \frac{19-9\sqrt{5}}{-11} = a\sqrt{5} - \frac{19}{11}$

$\Rightarrow \frac{9\sqrt{5}}{11} - \frac{19}{11} = a\sqrt{5} - \frac{19}{11}$

$\Rightarrow \frac{9\sqrt{5}}{11} = a\sqrt{5}$

Thus, $a = \frac{9}{11}$

(iii) We have, $\frac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}} = 2 - b\sqrt{6}$

By rationalising the denominator on L.H.S. of above equation, we get

$$\frac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}} \times \frac{3\sqrt{2}+2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}} = 2 - b\sqrt{6}$$

$$\Rightarrow \frac{\sqrt{2}(3\sqrt{2}+2\sqrt{3})+\sqrt{3}(3\sqrt{2}+2\sqrt{3})}{(3\sqrt{2})^2-(2\sqrt{3})^2} = 2 - b\sqrt{6}$$

$$\Rightarrow \frac{6+2\sqrt{6}+3\sqrt{6}+6}{18-12} = 2 - b\sqrt{6}$$

$$\Rightarrow \frac{12+5\sqrt{6}}{6} = 2 - b\sqrt{6}$$

$$\Rightarrow 2 + \frac{5\sqrt{6}}{6} = 2 - b\sqrt{6}$$

Thus, $b = -\frac{5}{6}$

(iv) We have, $\frac{7+\sqrt{5}}{7-\sqrt{5}} - \frac{7-\sqrt{5}}{7+\sqrt{5}} = a + \frac{7}{11}\sqrt{5}b$

$$\Rightarrow \frac{(7+\sqrt{5})^2-(7-\sqrt{5})^2}{(7-\sqrt{5})(7+\sqrt{5})} = a + \frac{7}{11}\sqrt{5}b$$

$$\Rightarrow \frac{49+5+14\sqrt{5}-49-5+14\sqrt{5}}{49-5} = a + \frac{7}{11}\sqrt{5}b$$

$$\Rightarrow \frac{28\sqrt{5}}{44} = a + \frac{7}{11}\sqrt{5}b$$

$$\Rightarrow 0 + \frac{7}{11}\sqrt{5} = a + \frac{7}{11}\sqrt{5}b$$

On comparing both sides, we get

$a = 0$ and $b = 1$

**Q12.**

**If $a = 2 + \sqrt{3}$, then find the value of $a - \frac{1}{a}$.**

**Solution:**

We have, $a = 2 + \sqrt{3}$ ...(i)

$$\Rightarrow \frac{1}{a} = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$$

[By rationalising the denominator]

$$= \frac{2-\sqrt{3}}{(2)^2-(\sqrt{3})^2} = \frac{2-\sqrt{3}}{4-3} = 2 - \sqrt{3}$$

$\Rightarrow \frac{1}{a} = 2 - \sqrt{3}$ ...(ii)

Thus, $a - \frac{1}{a} = (2+\sqrt{3}) - (2-\sqrt{3})$ [From (i) & (ii)]

$= 2\sqrt{3}$

**Q13.**

**Rationalise the denominator in each of the following and hence evaluate by taking $\sqrt{2}=1.414$, $\sqrt{3}=1.732$ and $\sqrt{5}=2.236$, upto three places of decimal.**

**(i)** $\frac{4}{\sqrt{3}}$ **(ii)** $\frac{6}{\sqrt{6}}$

**(iii)** $\frac{\sqrt{10}-\sqrt{5}}{2}$ **(iv)** $\frac{\sqrt{2}}{2+\sqrt{2}}$

**(v)** $\frac{1}{\sqrt{3}+\sqrt{2}}$

**Solution:**

(i) We have, $\frac{4}{\sqrt{3}} = \frac{4}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{4\sqrt{3}}{3} = \frac{4}{3} \times 1.732 = 2.309$

(ii) We have, $\frac{6}{\sqrt{6}} = \frac{6}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}}$

$= \frac{6\sqrt{6}}{6} = \sqrt{2} \times \sqrt{3} = 1.414 \times 1.732 = 2.449$

(iii) We have, $\frac{\sqrt{10}-\sqrt{5}}{2}$

$= \frac{\sqrt{5}\cdot\sqrt{2}-\sqrt{5}}{2} = \frac{\sqrt{5}(\sqrt{2}-1)}{2}$

$= \frac{2.236(1.414-1)}{2} = 0.46285 \approx 0.463$

(iv) We have, $\frac{\sqrt{2}}{2+\sqrt{2}} = \frac{\sqrt{2}}{2+\sqrt{2}} \times \frac{2-\sqrt{2}}{2-\sqrt{2}}$

$= \frac{\sqrt{2}(2-\sqrt{2})}{(2)^2-(\sqrt{2})^2} = \frac{\sqrt{2}\times\sqrt{2}(\sqrt{2}-1)}{2}$

$= \frac{2(\sqrt{2}-1)}{2} = \sqrt{2}-1$

$= 1.414 - 1 = 0.414$

(v) We have, $\frac{1}{\sqrt{3}+\sqrt{2}}$

$= \frac{1}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

$= \frac{\sqrt{3}-\sqrt{2}}{(\sqrt{3})^2-(\sqrt{2})^2} = \frac{\sqrt{3}-\sqrt{2}}{3-2}$

$= \sqrt{3}-\sqrt{2} = 1.732 - 1.414$

$= 0.318$

**Q14. Simplify**

(i) $(1^3+2^3+3^3)^{\frac{1}{2}}$ (ii) $\left(\frac{3}{5}\right)^4\left(\frac{8}{5}\right)^{-12}\left(\frac{32}{5}\right)^6$

(iii) $\left(\frac{1}{27}\right)^{\frac{-2}{3}}$ (iv) $\left[\left((625)^{-\frac{1}{2}}\right)^{-\frac{1}{4}}\right]^2$

(v) $\dfrac{9^{\frac{1}{3}}\times 27^{-\frac{1}{2}}}{3^{\frac{1}{6}}\times 3^{-\frac{2}{3}}}$ (vi) $64^{-\frac{1}{3}}\left[64^{\frac{1}{3}}-64^{\frac{2}{3}}\right]$

(vii) $\dfrac{8^{\frac{1}{3}}\times 16^{\frac{1}{3}}}{32^{-\frac{1}{3}}}$

**Solution:**

(i) We have, $(1^3+2^3+3^3)^{\frac{1}{2}}=(1+8+27)^{\frac{1}{2}}$

$=(36)^{\frac{1}{2}}=(6^2)^{\frac{1}{2}}=6$

(ii) We have, $\left(\frac{3}{5}\right)^4\left(\frac{8}{5}\right)^{-12}\left(\frac{32}{5}\right)^6$

$=\frac{3^4}{5^4}\times\left(\frac{5}{2^3}\right)^{12}\times\left(\frac{2^5}{5}\right)^6=\frac{3^4}{5^4}\times\frac{5^{12}}{2^{36}}\times\frac{2^{30}}{5^6}$

$=\frac{3^4}{2^6}\times 5^2=\frac{81\times 25}{64}=\frac{2025}{64}$

(iii) We have, $\left(\frac{1}{27}\right)^{-2/3}=\left(\frac{1}{3^3}\right)^{-2/3}=(3^3)^{2/3}$

$=3^2=9$

(iv) We have, $\left[\left((625)^{-\frac{1}{2}}\right)^{-\frac{1}{4}}\right]^2=\left[\left((25^2)^{-\frac{1}{2}}\right)^{-\frac{1}{4}}\right]^2$

$=(25^{-1})^{-\frac{1}{4}\times 2}=[(5^2)^{-1}]^{-\frac{1}{2}}=5$

(v) We have, $\dfrac{9^{\frac{1}{3}}\times 27^{-\frac{1}{2}}}{3^{\frac{1}{6}}\times 3^{-\frac{2}{3}}}=\dfrac{(3^2)^{\frac{1}{3}}\times(3^3)^{-\frac{1}{2}}}{3^{\frac{1}{6}}\times 3^{-\frac{2}{3}}}$

$=\dfrac{3^{\frac{2}{3}}\times 3^{-\frac{3}{2}}}{3^{\frac{1}{6}}\times 3^{-\frac{2}{3}}}=\dfrac{3^{\frac{2}{3}-\frac{3}{2}}}{3^{\frac{1}{6}-\frac{2}{3}}}=3^{-\frac{5}{6}+\frac{3}{6}}$

$=3^{-\frac{2}{6}}=3^{-\frac{1}{3}}$

(vi) We have, $64^{-\frac{1}{3}}\left[64^{\frac{1}{3}}-64^{\frac{2}{3}}\right]$

$=(4^3)^{-\frac{1}{3}}\left[(4^3)^{\frac{1}{3}}-(4^3)^{\frac{2}{3}}\right]$

$=4^{3\times\left(-\frac{1}{3}\right)}\left(4^{3\times\frac{1}{3}}-4^{3\times\frac{2}{3}}\right)$

$=4^{-1}(4-4^2)=\frac{1}{4}(4-16)=-\frac{12}{4}=-3$

(vii) $\dfrac{8^{\frac{1}{3}}\times 16^{\frac{1}{3}}}{32^{-\frac{1}{3}}}=\dfrac{(2^3)^{\frac{1}{3}}\times(2^4)^{\frac{1}{3}}}{(2^5)^{-\frac{1}{3}}}=\dfrac{2^{\frac{3}{3}+\frac{4}{3}}}{2^{-\frac{5}{3}}}$

$=2^{\frac{7}{3}+\frac{5}{3}}=2^{\frac{12}{3}}=2^4=16$

## Exercise 1.4: Long Answer Type Questions

**Q1.** Express 0.6+ 0.7¯ + 0.47¯ in the form p/q , where p and q are integers and q≠0

**Solution:**

Let $x = 0.\overline{7}$ or $x = 0.777...$ ...(i)

On multiplying both sides of (i) by 10, we get

$10x = 7.77...$ ...(ii)

On subtracting (i) from (ii), we get

$10x - x = (7.77...) - (0.77...) \Rightarrow 9x = 7$

Thus, $x = \frac{7}{9}$

Now, let $y = 0.4\overline{7}$ or $y = 0.4777...$ ...(iii)

On multiplying both sides of (iii) by 10, we get

$10y = 4.777...$ ...(iv)

On subtracting (iii) from (iv), we get

$10y - y = (4.777...) - (0.477...)$

$\Rightarrow 9y = 4.3$

Thus, $y = \frac{43}{90}$

$$\therefore \quad 0.6 + 0.\overline{7} + 0.4\overline{7} = \frac{6}{10} + \frac{7}{9} + \frac{43}{90}$$

$$= \frac{54 + 70 + 43}{90} = \frac{167}{90}$$

**Q2. Simplify**

$$\frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}}-\frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}}-\frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}.$$

**Solution:**

$$\frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}}=\frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}}\times\frac{\sqrt{10}-\sqrt{3}}{\sqrt{10}-\sqrt{3}}$$

[By rationalising the denominator]

$$=\frac{7(\sqrt{30}-3)}{(\sqrt{10})^2-(\sqrt{3})^2}$$

$$=\frac{7(\sqrt{30}-3)}{7}=\sqrt{30}-3 \qquad \text{...(i)}$$

Now, $$\frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}}=\frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}}\times\frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}-\sqrt{5}}$$

[by rationalising the denominator]

$$=\frac{2\sqrt{30}-10}{(\sqrt{6})^2-(\sqrt{5})^2}=\frac{2\sqrt{30}-10}{6-5}=2\sqrt{30}-10 \qquad \text{...(ii)}$$

and $$\frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}=\frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}\times\frac{\sqrt{15}-3\sqrt{2}}{\sqrt{15}-3\sqrt{2}}$$

[By rationalising the denominator]

$$=\frac{3(\sqrt{30}-6)}{(\sqrt{15})^2-(3\sqrt{2})^2}=\frac{3(\sqrt{30}-6)}{-3}$$

$$=6-\sqrt{30} \qquad \text{...(iii)}$$

$$\therefore \quad \frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}}-\frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}}-\frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}$$

$$=(\sqrt{30}-3)-(2\sqrt{30}-10)-(6-\sqrt{30})$$

[From (i), (ii) & (iii)]

$$=\sqrt{30}-3-2\sqrt{30}+10-6+\sqrt{30}=1$$

**Q3.**

**If $\sqrt{2} = 1.414$ and $\sqrt{3} = 1.732$, then find the value of $\frac{4}{3\sqrt{3} - 2\sqrt{2}} + \frac{3}{3\sqrt{3} + 2\sqrt{2}}$.**

**Solution:**

We have, $\frac{4}{3\sqrt{3} - 2\sqrt{2}} + \frac{3}{3\sqrt{3} + 2\sqrt{2}}$

$$= \frac{4(3\sqrt{3} + 2\sqrt{2}) + 3(3\sqrt{3} - 2\sqrt{2})}{(3\sqrt{3} - 2\sqrt{2})(3\sqrt{3} + 2\sqrt{2})}$$

$$= \frac{12\sqrt{3} + 8\sqrt{2} + 9\sqrt{3} - 6\sqrt{2}}{(3\sqrt{3})^2 - (2\sqrt{2})^2}$$

$$= \frac{21\sqrt{3} + 2\sqrt{2}}{27 - 8} = \frac{21\sqrt{3} + 2\sqrt{2}}{19}$$

$$= \frac{21 \times 1.732 + 2 \times 1.414}{19}$$

$$= \frac{39.2}{19} = 2.06316 \approx 2.063$$

**Q4.**

**If $a=\frac{3+\sqrt{5}}{2}$, then find the value of $a^2+\frac{1}{a^2}$.**

**Solution:**

We have, $a=\frac{3+\sqrt{5}}{2}$ ...(i)

Now, $\frac{1}{a}=\frac{2}{3+\sqrt{5}}=\frac{2}{3+\sqrt{5}}\times\frac{3-\sqrt{5}}{3-\sqrt{5}}$

[By rationalising the denominator]

$=\frac{6-2\sqrt{5}}{3^2-(\sqrt{5})^2}=\frac{6-2\sqrt{5}}{9-5}=\frac{6-2\sqrt{5}}{4}$

$\Rightarrow \frac{1}{a}=\frac{2(3-\sqrt{5})}{4}=\frac{3-\sqrt{5}}{2}$ ...(ii)

Consider $a^2+\frac{1}{a^2}=a^2+\frac{1}{a^2}+2-2=\left(a+\frac{1}{a}\right)^2-2$

$=\left(\frac{3+\sqrt{5}}{2}+\frac{3-\sqrt{5}}{2}\right)^2-2$ [From (i) and (ii)]

$=\left(\frac{6}{2}\right)^2-2=(3)^2-2=9-2=7$

**Q5.**

**If $x = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ and $y = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$, then find the value of $x^2 + y^2$.**

**Solution:**

We have $x = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

[By rationalising the denominator]

$= \frac{(\sqrt{3}+\sqrt{2})^2}{(\sqrt{3})^2-(\sqrt{2})^2} = 3+2+2\sqrt{6}$

So, $x = 5+2\sqrt{6}$ ...(i)

On squaring both sides, we get

$x^2 = (5+2\sqrt{6})^2$

$\Rightarrow x^2 = 25+24+20\sqrt{6}$

$\Rightarrow x^2 = 49+20\sqrt{6}$ ...(ii)

Now, $y = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} = \frac{1}{x} = \frac{1}{5+2\sqrt{6}}$ [From (i)]

$= \frac{1}{5+2\sqrt{6}} \times \frac{5-2\sqrt{6}}{5-2\sqrt{6}}$

[By rationalising the denominator]

$\Rightarrow y = \frac{5-2\sqrt{6}}{(5)^2-(2\sqrt{6})^2} = \frac{5-2\sqrt{6}}{25-24} = 5-2\sqrt{6}$

On squaring both sides, we get

$y^2 = (5-2\sqrt{6})^2$

$\Rightarrow y^2 = 25+24-20\sqrt{6}$

$\Rightarrow y^2 = 49-20\sqrt{6}$ ...(iii)

On adding (ii) and (iii), we get

$x^2+y^2 = 49+20\sqrt{6}+49-20\sqrt{6} = 98$

**Q6.**

**Simplify:** $(256)^{-\left(4^{\frac{-3}{2}}\right)}$

**Solution:**

We have, $(256)^{-\left(4^{\frac{-3}{2}}\right)} = (256)^{-x}$, where

$$x = 4^{\left(\frac{-3}{2}\right)} = (2)^{2\times\left(\frac{-3}{2}\right)} = 2^{-3} = \frac{1}{2^3} = \frac{1}{8} \qquad \text{...(i)}$$

$$\text{Now } (256)^{-\left(4^{\frac{-3}{2}}\right)} = (2^8)^{\frac{-1}{8}} \qquad \text{[From (i)]}$$

$$= 2^{-1} = \frac{1}{2}$$

**Q7.**

**Find the value of** $\frac{4}{(216)^{-\frac{2}{3}}} + \frac{1}{(256)^{-\frac{3}{4}}} + \frac{2}{(243)^{-\frac{1}{5}}}$.

**Solution:**

We have, $\frac{4}{(216)^{-\frac{2}{3}}} + \frac{1}{(256)^{-\frac{3}{4}}} + \frac{2}{(243)^{-\frac{1}{5}}}$

$$= \frac{4}{(6^3)^{-\frac{2}{3}}} + \frac{1}{(4^4)^{-\frac{3}{4}}} + \frac{2}{(3^5)^{-\frac{1}{5}}}$$

$$= \frac{4}{6^{-2}} + \frac{1}{4^{-3}} + \frac{2}{3^{-1}}$$

$$= 4\times 6^2 + 4^3 + 2\times 3^1$$

$$= 144 + 64 + 6 = 214$$