Exercise 8.1

Q1. The angles of quadrilateral are in the ratio 3 : 5 : 9 : 13. Find all the angles of the quadrilateral.

Answer:

The sum of interior angles in a quadrilateral is 360 degrees.

Let the angles of the quadrilateral be 3x, 5x, 9x, and 13x respectively.

The sum of all interior angles of a quadrilateral is 360°.

$\therefore 3x + 5x + 9x + 13x = 360°$

$30x = 360°$

$x = 12°$

Hence, the angles are

$3x = 3 \times 12 = 36°$

$5x = 5 \times 12 = 60°$

$9x = 9 \times 12 = 108°$

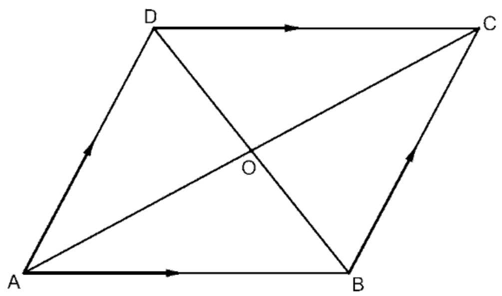
$13x = 13 \times 12 = 156°$

Q2. If the diagonals of a parallelogram are equal, then show that it is a rectangle

Answer:

Given: The diagonals of a parallelogram are equal.

To show that a given parallelogram is a rectangle, we have to prove that one of its interior angles is 90° and this can be done by the concept of congruent triangles.



Let ABCD be a parallelogram. To show that ABCD is a rectangle, we have to prove that one of its interior angles is 90°.

In ΔABC and ΔDCB,

AB = DC (Opposite sides of a parallelogram are equal)

BC = BC (Common)

AC = DB (Given the diagonals are equal)

∴ ΔABC ≅ ΔDCB (By SSS Congruence rule)

⇒ ∠ABC = ∠DCB (By CPCT) ------------- (1)

It is known that the sum of the measures of angles on the same side of transversal is 180° (co - interior angles)

∠ABC + ∠DCB = 180° (AB || CD)

⇒∠ABC + ∠ABC = 180° [From equation(1)]

⇒ 2∠ABC = 180°

⇒∠ABC = 90°

Thus, ∠DCB = 90° [From equation (1)]

Hence, ∠B = ∠D = ∠C = ∠A = 90° [Since opposite angles of a parallelogram are equal].

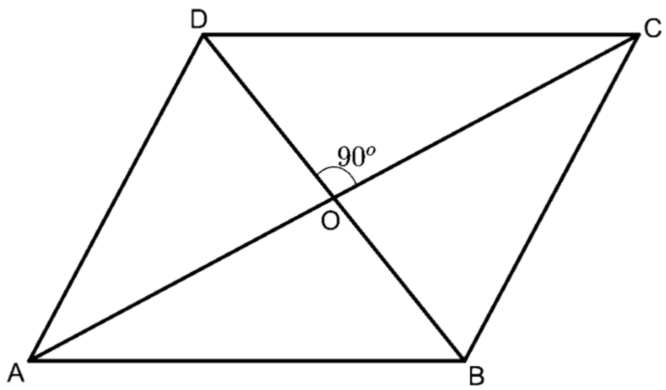
Since ABCD is a parallelogram and one of its interior angles is 90°, ABCD is a rectangle.

Q3. Show that if the diagonals of a quadrilateral bisect each other at right angles, then it is a rhombus

Answer:

Given: The diagonals of a quadrilateral bisect each other at right angles.

To show that a given quadrilateral is a rhombus, we have to show it is a parallelogram and all the sides are equal.



Let ABCD be a quadrilateral, whose diagonals AC and BD bisect each other at the right angle.

So, we have, OA = OC, OB = OD, and ∠AOB = ∠BOC = ∠COD = ∠AOD = 90°.

To prove ABCD a rhombus, we have to prove ABCD is a parallelogram and all the sides of ABCD are equal.

In ΔAOD and ΔCOD,

OA = OC (Diagonals bisect each other)

∠AOD = ∠COD = 90° (Given)

OD = OD (Common)

∴ ΔAOD ≅ ΔCOD (By SAS congruence rule)

∴ AD = CD (By CPCT) ....(1)

Similarly, it can be proved that

AD = AB and CD = BC ....(2)

From Equations (1) and (2), AB = BC = CD = AD

Since opposite sides of quadrilateral ABCD are equal, it can be said that ABCD is a parallelogram. Since all sides of a parallelogram ABCD are equal, it can be said that ABCD is a rhombus.

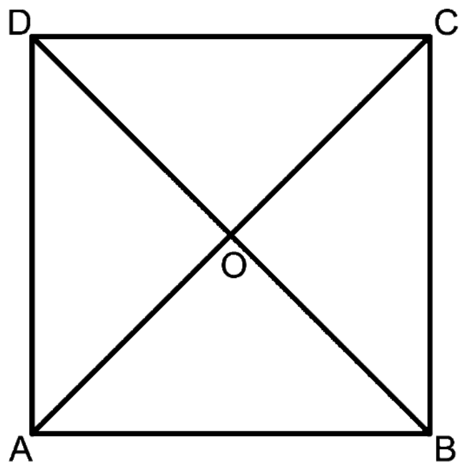
Q4. Show that the diagonals of a square are equal and bisect each other at right angles

Answer:

Given: The quadrilateral is a square.

To prove: Diagonals of a square are equal and bisect each other at right angles.

Thus, we have to prove AC = BD, OA = OC, OB = OD, and ∠AOB = 90°



Let ABCD be a square.

Let the diagonals AC and BD intersect each other at a point O.

In ΔABC and ΔDCB,

AB = DC (Sides of a square are equal to each other)

∠ABC = ∠DCB (All interior angles are of 90°)

BC = CB (Common side)

∴ ΔABC ≅ ΔDCB (By SAS congruence rule)

∴ AC = DB (By CPCT) ------------ (1)

Hence, the diagonals of a square are equal in length.

In ΔAOB and ΔCOD,

∠AOB = ∠COD (Vertically opposite angles)

∠ABO = ∠CDO (Alternate interior angles)

AB = CD (Sides of a square are always equal)

∴ ΔAOB ≅ ΔCOD (By AAS congruence rule)

∴ AO = CO and OB = OD (By CPCT) ------------ (2)

Hence, the diagonals of a square bisect each other.

In ΔAOB and ΔCOB,

As we had proved that diagonals bisect each other,

Therefore, AO = CO

AB = CB (Sides of a square are equal)

BO = BO (Common)

∴ ΔAOB ≅ ΔCOB (By SSS congruency)

∴ ∠AOB = ∠COB (By CPCT) --------- (3)

However, ∠AOB + ∠COB = 180° (Linear pair)

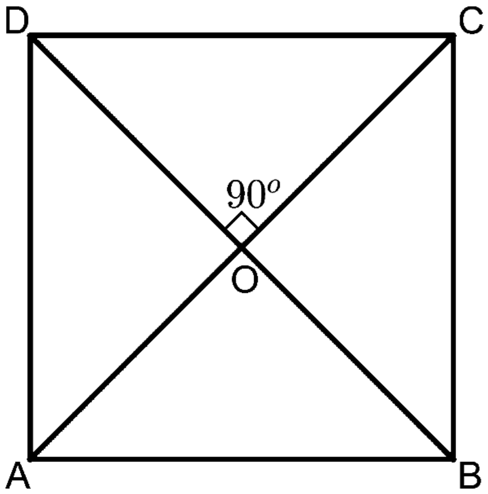
2∠AOB = 180° [From equation (3)]

∠AOB = 90° ------------- (4)

Hence, from equation (1), (2) and (4), we see that the diagonals of a square are equal and bisect each other at right angles

Q5. Show that if the diagonals of a quadrilateral are equal and bisect each other at right angles, then it is a square.

Answer: We have to show that a given quadrilateral is a parallelogram in which all sides are equal and one of its interior angles is 90°.



Let us consider a quadrilateral ABCD in which the diagonals AC and BD intersect each other at O.

It is given that the diagonals of ABCD are equal and bisect each other at right angles.

Therefore, AC = BD, OA = OC, OB = OD, and ∠AOB = ∠BOC = ∠COD = ∠AOD = 90°

To prove ABCD is a square, we have to prove that ABCD is a parallelogram in which AB = BC = CD = AD, and one of its interior angles is 90°.

In ΔAOB and ΔCOD,

AO = CO (Diagonals bisect each other)

OB = OD (Diagonals bisect each other)

∠AOB = ∠COD (Vertically opposite angles)

∴ ΔAOB ≅ ΔCOD (SAS congruence rule)

∴ AB = CD (By CPCT) .... (1)

And, ∠OAB = ∠OCD (By CPCT)

However, these are alternate interior angles for line AB and CD and alternate interior angles are equal to each other only when the two lines are parallel.

Thus, AB || CD .... (2)

From Equations (1) and (2), we obtain ABCD is a parallelogram.

In ΔAOD and ΔCOD,

AO = CO (Diagonals bisect each other)

∠AOD = ∠COD (Each angle is 90°)

OD = OD (Common)

∴ ΔAOD ≅ ΔCOD (SAS congruence rule)

∴ AD = DC (By CPCT) .... (3)

However, AD = BC and AB = CD (Opposite sides of parallelogram ABCD are equal)

∴ AB = BC = CD = DA

Therefore, all the sides of quadrilateral ABCD are equal to each other.

In ΔADC and ΔBCD,

AD = BC (Already proved)

AC = BD (Given)

DC = CD (Common)

∴ ΔADC ≅ ΔBCD (SSS Congruence rule)

∴ ∠ADC = ∠BCD (By CPCT)

However, ∠ADC + ∠BCD = 180°(Co-interior angles)

∠ADC + ∠ADC = 180°

2∠ADC = 180°

∴ ∠ADC = 90°

One of the interior angles of quadrilateral ABCD is a right angle.

Thus, we have obtained that ABCD is a parallelogram where AB = BC = CD = AD and one of its interior angles is 90°.

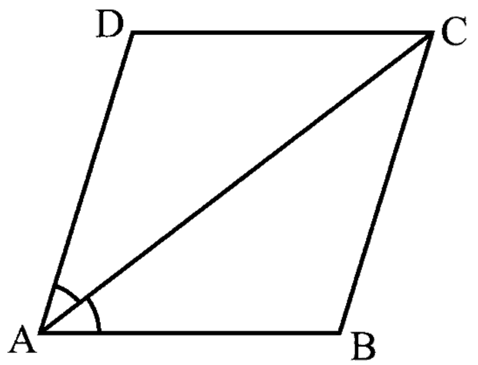
Therefore, ABCD is a square.

Q6. Diagonal AC of a parallelogram ABCD bisects ∠A (see Fig. 8.19). Show that

i) it bisects ∠C also, ii) ABCD is a rhombus

Answer: The diagonal AC of a parallelogram ABCD bisects ∠A.

We can use alternate interior angles property to show that the diagonal AC bisects ∠C and by showing all sides are equal, it can be proved ABCD is a rhombus.



i) ABCD is a parallelogram.

∠DAC = ∠BCA (Alternate interior angles) ....(1)

∠BAC = ∠DCA (Alternate interior angles) ....(2)

However, it is given that AC bisects ∠A.

∠DAC = ∠BAC ....(3)

From equations (1), (2), and (3), we obtain

∠DCA = ∠BAC = ∠DAC = ∠BCA ....(4)

Thus, ∠DCA = ∠BCA

Hence, AC bisects ∠C.

ii) From Equation (4), we obtain

∠DAC = ∠DCA

DA = DC (Side opposite to equal angles are equal)

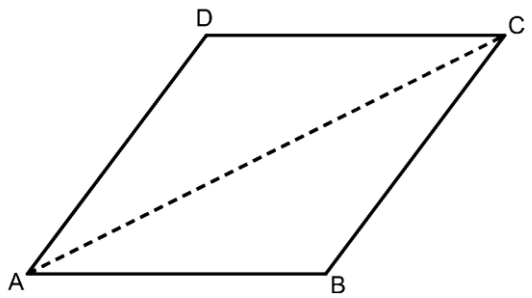
However, DA = BC and AB = CD (Opposite sides of a parallelogram are equal)

Thus, AB = BC = CD = DA

Hence, ABCD is a rhombus.

Q7. ABCD is a rhombus. Show that diagonal AC bisects ∠A as well as ∠C and diagonal BD bisects ∠B as well as ∠D.

Answer: Given: ABCD is a rhombus.



Let us join AC.

In ΔABC, BC = AB (Sides of a rhombus are equal to each other)

So, ∠BAC = ∠BCA (Angles opposite to equal sides of a triangle are equal) ----------- (1)

However,

∠BAC = ∠DCA (Alternate interior angles for parallel lines AB and CD) -------------- (2)

From equation (1) and (2), ∠BCA = ∠DCA

Therefore, AC bisects ∠C.

Also, ∠BCA = ∠DAC (Alternate interior angles for parallel lines BC and DA)

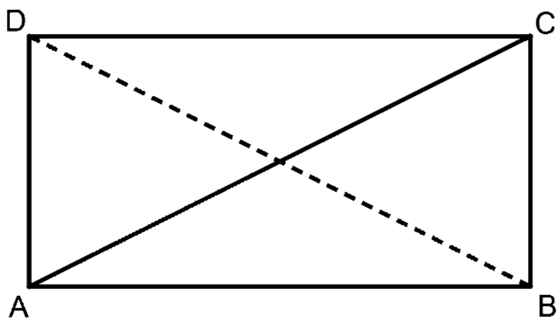
Thus, ∠BAC = ∠DAC

Therefore, AC bisects ∠A as well as ∠C.

Similarly, it can be proved that BD bisects ∠B and ∠D as well.

Q8. ABCD is a rectangle in which diagonal AC bisects ∠A as well as ∠C. Show that:
(i) ABCD is a square (ii) diagonal BD bisects ∠B as well as ∠D.

Answer:



(i) We are given that ABCD is a rectangle, so

∠A = ∠C

⇒ 1/2 ∠A = 1/2 ∠C

⇒ ∠DAC = ∠DCA (Given that AC bisects ∠A and ∠C)

Thus, CD = DA (Sides opposite to equal angles are also equal)

However, DA = BC and AB = CD (Opposite sides of a rectangle are equal)

Thus AB = BC = CD = DA

ABCD is a rectangle and all the sides are equal. Hence, ABCD is a square.

(ii) Let us join BD

Since ABCD is square, AB || CD and BC || AD

In ΔBCD,

BC = CD (Sides of a square are equal to each other)

∠CDB = ∠CBD (Angles opposite to equal sides are equal) ..... (1)

However, ∠CDB = ∠ABD (Alternate interior angles as AB || CD) ..... (2)

From equations (1) and (2), ∠CBD = ∠ABD

Thus, BD bisects ∠B.

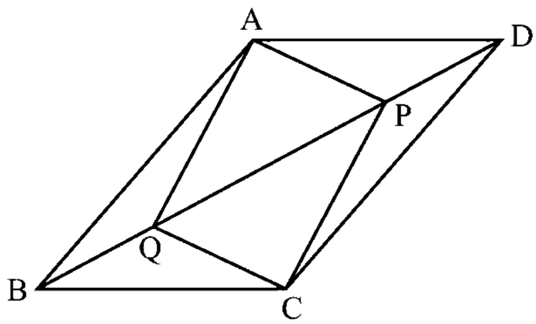
Also, ∠CBD = ∠ADB (Alternate interior angles for BC || AD) .....(3)

So, using equations (1) and (3), ∠ADB = ∠CDB

Hence, BD bisects ∠D and ∠B.

Q9. In parallelogram ABCD, two points P and Q are taken on diagonal BD such that DP = BQ (see Fig. 8.20). Show that:
(i) Δ APD ≅ Δ CQB
(ii) AP = CQ
(iii) Δ AQB ≅ Δ CPD
(iv) AQ = CP
(v) APCQ is a parallelogram

Answer:



Given: ABCD is a parallelogram and DP = BQ

(i) In ΔAPD and ΔCQB,

∠ADP = ∠CBQ (Alternate interior angles for BC || AD)

AD = CB (Opposite sides of parallelogram ABCD)

DP = BQ (Given)

∴ ΔAPD ≅ ΔCQB (Using SAS congruence rule)

(ii) Since ΔAPD ≅ ΔCQB,

∴ AP = CQ (By CPCT)

(iii) In ΔAQB and ΔCPD,

AB = CD (Opposite sides of parallelogram ABCD)

∠ABQ = ∠CDP (Alternate interior angles for AB || CD)

BQ = DP (Given)

∴ ΔAQB ≅ ΔCPD (Using SAS congruence rule)

(iv) Since ΔAQB ≅ ΔCPD,

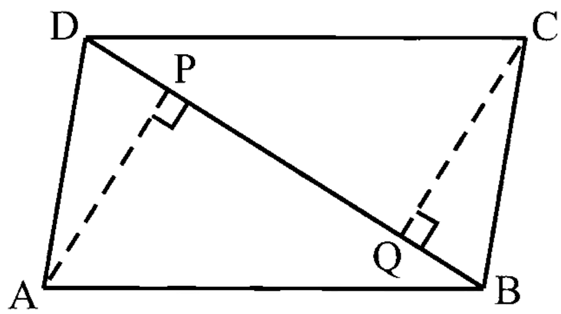
∴ AQ = CP (CPCT)

(v) From the result obtained in (ii) and (iv), AQ = CP and AP = CQ

Since opposite sides in quadrilateral APCQ are equal to each other, thus APCQ is a parallelogram.

Q10. ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on diagonal BD (see Fig. 8.21). Show that (i) ΔAPB ≅ ΔCQD (ii) AP = CQ

Answer:



Given: ABCD is a parallelogram and AP ⊥ DB, CQ ⊥ DB

(i) In ΔAPB and ΔCQD,

∠APB = ∠CQD (Each 90°)

AB = CD (Opposite sides of parallelogram ABCD)

∠ABP = ∠CDQ (Alternate interior angles as AB || CD)

∴ ΔAPB ≅ ΔCQD (By AAS congruency)

(ii) By using the result ΔAPB ≅ ΔCQD., we obtain AP = CQ (By CPCT)

Q11. In △ABC and △DEF, AB = DE, AB || DE, BC = EF and BC || EF. Vertices A, B and C are joined to vertices D, E and F respectively (see Fig. 8.22). Show that

(i) quadrilateral ABED is a parallelogram

(ii) quadrilateral BEFC is a parallelogram

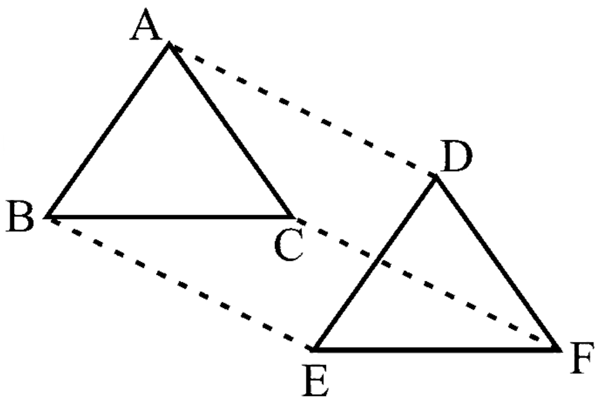
(iii) AD || CF and AD = CF

(iv) quadrilateral ACFD is a parallelogram

(v) AC = DF

(vi) △ABC ≅ △DEF.

Answer:



Given: In ΔABC and ΔDEF, AB = DE, AB || DE, BC = EF and BC || EF.

We can use the fact that in a quadrilateral if one pair of opposite sides are parallel and equal to each other then it will be a parallelogram.

(i) It is given that AB = DE and AB || DE

If one pair of opposite sides of a quadrilateral are equal and parallel to each other, then it will be a parallelogram.

Therefore, quadrilateral ABED is a parallelogram.

(ii) It is given that BC = EF and BC || EF

Therefore, quadrilateral BEFC is a parallelogram.

(iii) As we had observed that ABED and BEFC are parallelograms, therefore

AD = BE and AD || BE (Opposite sides of a parallelogram are equal and parallel)

BE = CF and BE || CF (Opposite sides of a parallelogram are equal and parallel)

Thus, AD = BE = CF and AD || BE || CF

∴ AD = CF and AD || CF (Lines parallel to the same line are parallel to each other)

(iv) As we had observed that one pair of opposite sides (AD and CF) of quadrilateral ACFD are equal and parallel to each other, therefore, it is a parallelogram.

(v) As ACFD is a parallelogram, therefore, the pair of opposite sides will be equal and parallel to each other

∴ AC || DF and AC = DF

(vi) ΔABC and ΔDEF,

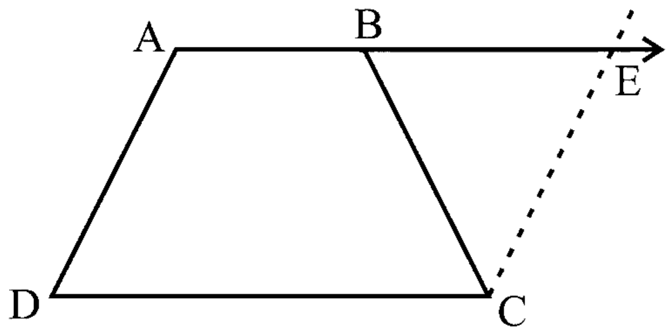
AB = DE (Given)

BC = EF (Given)

AC = DF (Since ACFD is a parallelogram)

∴ ΔABC ≅ ΔDEF (By SSS congruence rule)

Q12. ABCD is a trapezium in which AB || CD and AD = BC (see Fig. 8.23). Show that
(i) ∠A = ∠B
(ii) ∠C = ∠D
(iii) ΔABC ≅ ΔBAD
(iv) diagonal AC = diagonal BD
[Hint: Extend AB and draw a line through C parallel to DA intersecting AB produced at E.]



Solution:

Let us join BD and AC in the figure as shown below.. ADCE is a parallelogram.

(i) AD = CE (Opposite sides of parallelogram AECD are equal)

However, AD = BC (Given)

Therefore, BC = CE

∠CEB = ∠CBE (Angles opposite to equal sides in a triangle are also equal)

Consider, parallel lines AD and CE where AE is the transversal.

∠BAD + ∠CEB = 180° [Co-Interior angles]

∠BAD + ∠CBE = 180° ... (1) [Since, ∠CEB = ∠CBE]

However, ∠ABC + ∠CBE = 180° (Linear pair angles) ... (2)

From Equations (1) and (2), we see that

∠BAD = ∠ABC

Thus, ∠A = ∠B

(ii) AB || CD

∠A + ∠D = 180° (Angles on the same side of the transversal)

Also, ∠C + ∠B = 180° (Angles on the same side of the transversal)

∴ ∠A + ∠D = ∠C + ∠B

However, ∠A = ∠B [Using the result obtained in (i)]

∴ ∠C = ∠D

(iii) In ΔABC and ΔBAD,

AB = BA (Common side)

BC = AD (Given)

∠B = ∠A (Proved before)

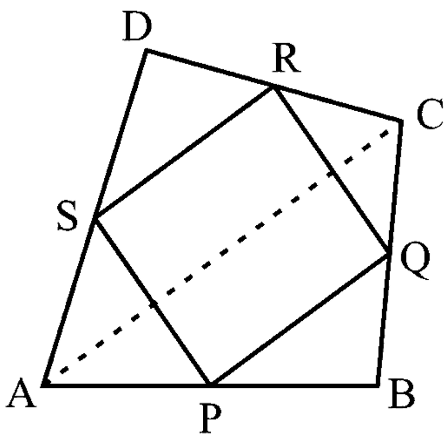
∴ ΔABC ≅ ΔBAD (SAS congruence rule)

(iv) Since ΔABC ≅ ΔBAD,

∴ AC = BD (By CPCT)

Exercise 8.2

Q1. ABCD is a quadrilateral in which P, Q, R and S are mid-points of the sides AB, BC, CD and DA (see Fig. 8.29). AC is a diagonal. Show that:
(i) SR || AC and SR = 1/2AC
(ii) PQ = SR
(iii) PQRS is a parallelogram.



Answer: We will use the mid-point theorem here. It that states that the line segment joining the mid-points of any two sides of the triangle is parallel to the third side and is half of it.

(i) In ΔADC, S and R are the mid-points of sides AD and CD respectively. Thus, by using the mid-point theorem

∴ SR || AC and SR = 1/2AC ... (1)

(ii) In ΔABC, P and Q are mid-points of sides AB and BC. Therefore, by using the mid-point theorem,

PQ || AC and PQ = 1/2 AC ... (2)

Using Equations (1) and (2), we obtain PQ || SR and PQ = SR ... (3)

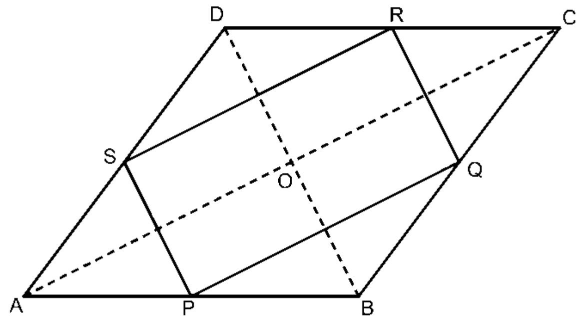
∴ PQ = SR

(iii) From Equation (3), we obtained PQ || SR and PQ = SR

Clearly, one pair of opposite sides of quadrilateral PQRS is parallel and equal. Hence, PQRS is a parallelogram.

Q2. ABCD is a rhombus and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rectangle.

Answer: We will use the mid-point theorem here. It states that the line segment joining the mid-points of any two sides of the triangle is parallel to the third side and is half of it.



In ΔABC, P and Q are the mid-points of sides AB and BC respectively.

∴ PQ || AC and PQ = 1/2 AC (Using mid-point theorem) ... (1)

In ΔADC,

R and S are the mid-points of CD and AD respectively.

∴ RS || AC and RS = 1/2 AC (Using mid-point theorem) ... (2)

From Equations (1) and (2), we obtain

PQ || RS and PQ = RS

Since in quadrilateral PQRS, one pair of opposite sides is equal and parallel to each other, it is a parallelogram.

Since the sides of a rhombus are equal, AB = BC

1/2 × AB = 1/2 × BC

PB = BQ (P and Q are the mid-points of sides AB and BC respectively)

∠QPB = ∠PQB (Sides opposite to equal angles are equal) ..... (3)

In ΔAPS and ΔCQR,

AP = CQ (P and Q are the mid-points of sides AB and BC respectively)

AS = CR (S and R are the mid-points of sides AD and CD respectively)

PS = QR (Opposite sides of a parallelogram are equal)

BY SSS congruency, ΔAPS ≅ ΔCQR

So, ∠APS = ∠CQR (By CPCT) .... (4)

Since AB is a straight line, ∠APS + ∠SPQ + ∠QPB = 180˚

Since BC is a straight line, ∠PQB + ∠PQR + ∠CQR = 180˚

∠APS + ∠SPQ + ∠QPB = ∠PQB + ∠PQR + ∠CQR

∠APS + ∠SPQ + ∠QPB = ∠QPB + ∠PQR + ∠APS (By equations (3) and (4))

∠SPQ = ∠PQR .... (5)

Since ∠SPQ and ∠PQR are interior angles on the same side of the transversal PQ, they form a pair of supplementary angles.

∠SPQ + ∠PQR = 180°

2∠SPQ = 180° [From (5)]

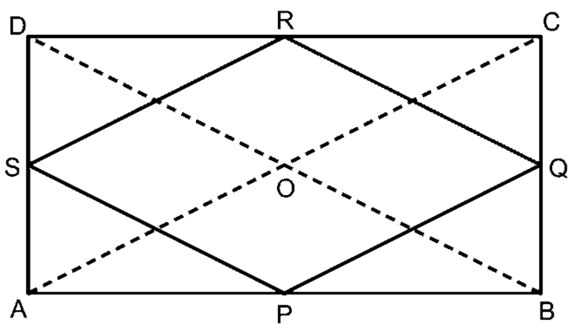
∠SPQ = 90°

Clearly, PQRS is a parallelogram having one of its interior angles as 90°.

Hence, PQRS is a rectangle.

Q3. ABCD is a rectangle and P, Q, R and S are mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rhombus.

Answer: We will use the mid-point theorem here. It states that the line segment joining the mid-points of any two sides of the triangle is parallel to the third side and is half of it.



Let us join AC and BD. In △ABC,

P and Q are the mid-points of AB and BC respectively.

∴ PQ || AC and PQ = 1/2 AC (Mid-point theorem) ... (1)

Similarly, in △ADC,

SR || AC and SR = 1/2 AC (Mid-point theorem) ... (2)

Clearly, PQ || SR and PQ = SR [From equation (1) and (2)]

Since in quadrilateral PQRS, one pair of opposite sides are equal and parallel to each other, it is a parallelogram.

∴ PS || QR and PS = QR (Opposite sides of the parallelogram) ... (3)

In △BCD, Q and R are the mid-points of side BC and CD respectively.

∴ QR || BD and QR = 1/2 BD (Mid-point theorem) ... (4)

However, the diagonals of a rectangle are equal.

∴ AC = BD ... (5)

Thus, QR = 1/2 AC

Also, in △BAD

PS || BD and PS = 1/2 BD

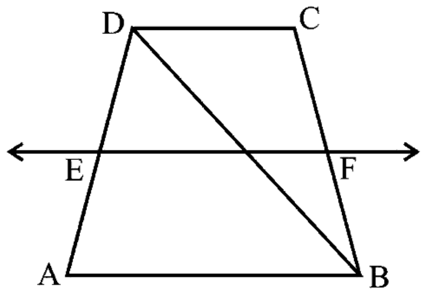
Thus, QR = PS .... (6)

By using Equations (1), (2), (3), (4), and (5), we obtain

PQ = QR = SR = PS

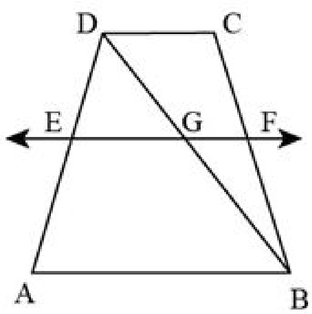
Therefore, PQRS is a rhombus.

Q4. ABCD is a trapezium in which AB || DC, BD is a diagonal and E is the mid-point of AD. A line is drawn through E parallel to AB intersecting BC at F (see Fig. 8.30). Show that F is the mid-point of BC.



Answer:

Let EF intersect DB at G as shown below.



By converse of mid-point theorem, we know that a line drawn through the mid-point of any side of a triangle and parallel to another side bisects the third side.

In trapezium ABCD,

EF || AB and E is the mid-point of AD.

Therefore, G is the mid-point of DB. [Converse of mid-point theorem]

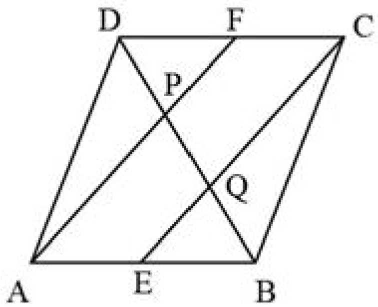
As EF || AB and AB || CD,

∴ EF || CD (Two lines parallel to the same line are parallel to each other)

In ΔBCD, GF || CD and G is the mid-point of line BD.

Therefore, by using the converse of mid-point theorem, F is the mid-point of BC.

Q5. In a parallelogram ABCD, E and F are the mid-points of sides AB and CD respectively (see Fig.8.31). Show that the line segments AF and EC trisect the diagonal BD.



Answer: In a parallelogram ABCD, E and F are the mid-points of sides AB and CD respectively.

To Prove: Line segments AF and EC trisect the diagonal BD.

According to the converse of mid-point theorem, we know that a line drawn through the mid-point of any side of a triangle that is parallel to another side bisects the third side.

ABCD is a parallelogram.

AB || CD

Hence, AE || FC

Again, AB = CD (Opposite sides of parallelogram ABCD)

⇒ 1/2 AB = 1/2 CD

⇒ AE = FC (E and F are mid-points of side AB and CD)

In quadrilateral AECF, one pair of opposite sides (AE and CF) is parallel and equal to each other.

Therefore, AECF is a parallelogram.

∴ AF || EC (Opposite sides of a parallelogram)

In ΔDQC, F is the mid-point of side DC and FP ||CQ (as AF || EC).

Therefore, by using the converse of the mid-point theorem, it can be said that P is the mid-point of DQ.

∴ DP = PQ --------------- (1)

Similarly, in ΔAPB, we know E is the mid-point of side AB and thus, EQ || AP (as AF || EC).

Therefore, by using the converse of the mid-point theorem, it can be said that Q is the mid-point of PB.

∴ PQ = QB --------------- (2)

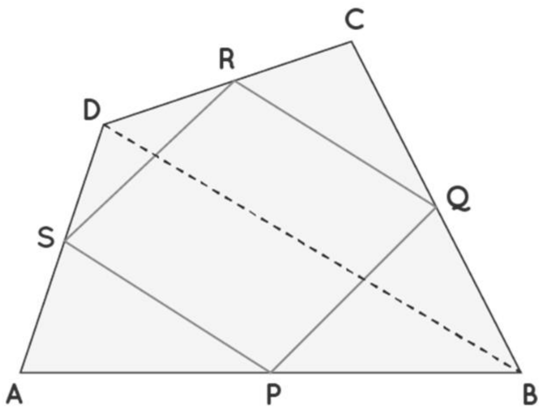
From equations (1) and (2),

DP = PQ = BQ

Hence, the line segments AF and EC trisect the diagonal BD.

Q6. Show that the line segments joining the mid-points of the opposite sides of a quadrilateral bisect each other.

Answer: In a triangle, the line segment joining the mid-points of any two sides of the triangle is parallel to the third side and is half of it.



Let ABCD is a quadrilateral in which P, Q, R, and S are the mid-points of sides AB, BC, CD, and DA respectively. Join PQ, QR, RS, SP, and BD.

In ΔABD, S and P are the mid-points of AD and AB respectively.

Therefore, by using the mid-point theorem, it can be said that

SP || BD and SP = 1/2 BD ---------- (1)

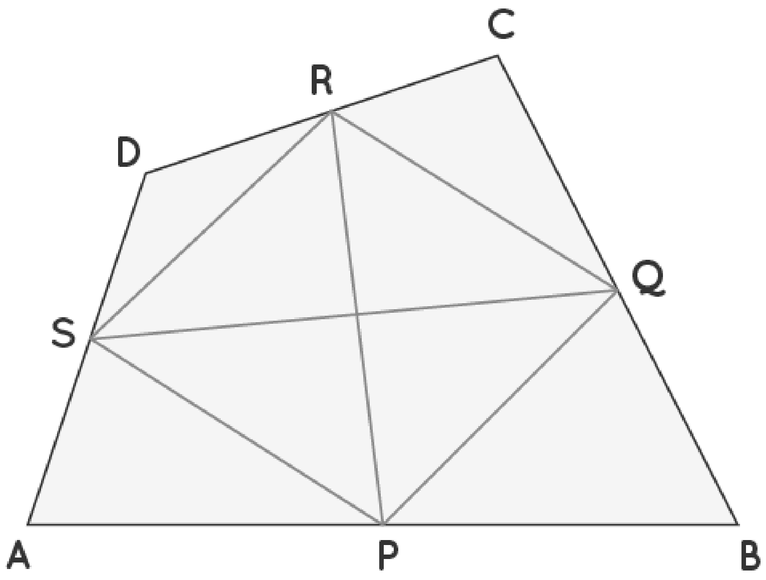
Similarly, in ΔBCD,

QR || BD and QR = 1/2 BD ---------- (2)

From equations (1) and (2), we obtain

SP || QR and SP = QR

In quadrilateral SPQR, one pair of opposite sides is equal and parallel to each other. Thus, SPQR is a parallelogram.



Since we know that diagonals of a parallelogram bisect each other we can conclude that PR and QS bisect each other as shown in the above figure.

Thus, we see that the line segments joining the mid-points of the opposite sides of a quadrilateral bisect each other.

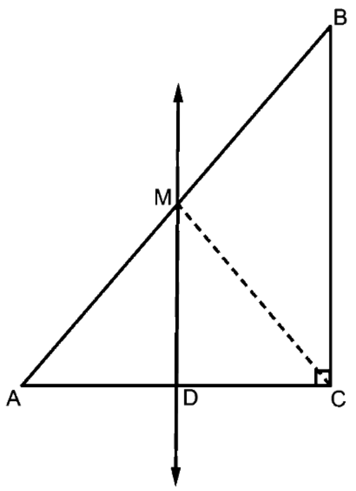
Q7. ABC is a triangle right angled at C. A line through the mid-point M of hypotenuse AB and parallel to BC intersects AC at D. Show that

(i) D is the mid-point of AC (ii) MD ⊥ AC

(iii) CM = MA = 1/2 AB

Answer: By converse of mid-point theorem, we know that a line drawn through the mid-point of any side of a triangle that is parallel to another side bisects the third side.

Let's construct the triangle according to the question.



(i) In ΔABC,

It is given that M is the mid-point of AB and MD || BC.

∴ D is the mid-point of AC. [Converse of mid-point theorem]

(ii) As DM || CB and AC is a transversal,

∠MDC + ∠DCB = 180° [Co-interior angles]

∠MDC + 90° = 180°

∠MDC = 90°

∴ MD ⊥ AC

(iii) Join MC

In ΔAMD and ΔCMD,

AD = CD (D is the mid-point of side AC)

∠ADM = ∠CDM (Each 90°)

DM = DM (Common)

∴ ΔAMD ≅ ΔCMD (By SAS congruence rule)

Therefore, AM = CM (By CPCT)

However, we also know that AM = 1/2 AB (M is the mid-point of AB)

∴ CM = AM = 1/2 AB