Exercise 5.1

Q1. Which of the following statements are true and which are false? Give reasons for your answers.

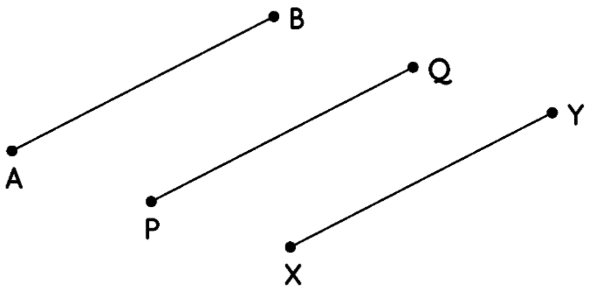
i) Only one line can pass through a single point.

ii) There are an infinite number of lines which pass through two distinct points.

iii) A terminated line can be produced indefinitely on both the sides.

iv) If two circles are equal, then their radii are equal.

v) In fig. 5.9, if AB = PQ and PQ = XY, then AB = XY.

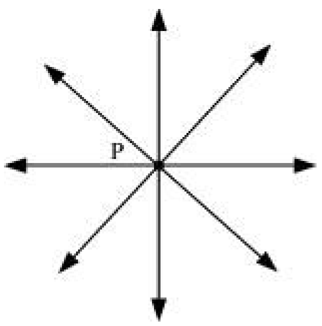


Answer:

i) Only one line can pass through a single point.

False because, we can draw infinite number of lines through a given point as shown below.

We can draw only one line passing through two distinct points as shown below.



ii) There are an infinite number of lines which pass through two distinct points.

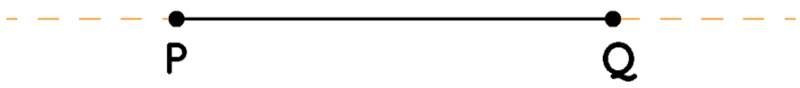
False. According to Axiom 5.1, Given any two distinct points, there is a unique line that passes through them.

We can draw only one line passing through two distinct points as shown below.

iii) A terminated line can be produced indefinitely on both the sides.

True. According to Postulate 2, a terminated line can be produced indefinitely.

We know that a straight line can be produced on both sides as shown using the dotted lines shown below.



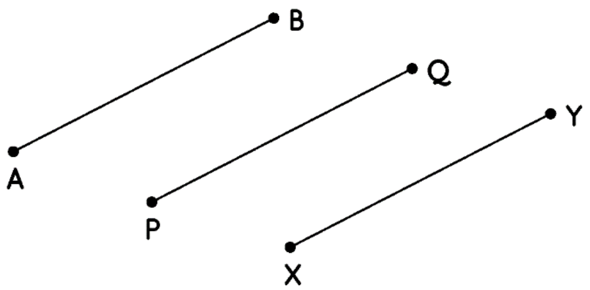
iv) If two circles are equal, then their radii are equal.

True. According to Postulate 3, A circle can be drawn with any center and any radius.

We know that, circles are equal, which means the circles are congruent. (Circles coinciding with each other). This means that circumferences are equal and so the radii of two circles are also equal.

v) In fig. 5.9, if AB = PQ and PQ = XY, then AB = XY.

True. Line segments whose corresponding lengths are equal are equal to one another.



By transitivity law, we know that, if a = b and b = c then a = c.

Here since AB = PQ and PQ = XY therefore, AB = XY according to this law.

Let us consider AB = 5 cm, then PQ will be 5 cm.

But PQ = XY so, XY will also be 5 cm.

Thus, AB = PQ = XY which also implies AB = XY.

Q2. Give a definition for each of the following terms. Are there other terms that need to be defined first? What are they, and how might you define them?
i) Parallel lines ii) Perpendicular lines iii) Line Segment
iv) Radius of a circle v) Square

Answer:

We have to define 'Ray', 'Straight line' and a 'point'.

Ray: A part of a line, which starts at a point (Here A) and goes off in a particular direction to infinity possibly through a second point (B in this case).

Straight Line: The basic concept about a line is that it should be straight, and it can be extended infinitely in both directions.

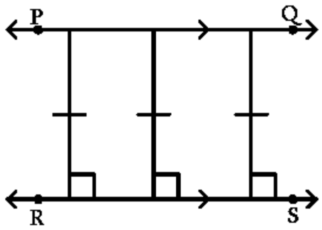
Point: A small dot made by a sharp pencil on a sheet of paper gives an idea about a point.

A point has no dimension i.e, length, breadth or height, it has only a position.

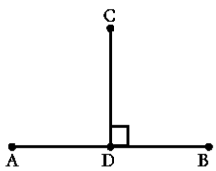
Perpendicular Distance: Shortest distance between a point and a line is known as perpendicular distance.

i) Parallel lines

- If the perpendicular distance between two lines is always constant, then these are called parallel.
- If the lengths of the common perpendiculars at different points on the lines are the same, then these lines are called parallel.
- In other words, the lines which never intersect each other are called parallel lines.



ii) Perpendicular lines: If the angle between two lines is equal to 90º, then these lines are perpendicular to each other.

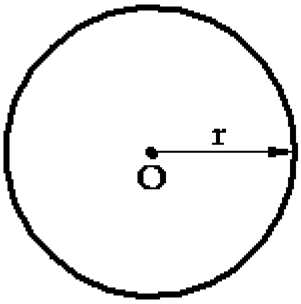


CD ⊥ AB

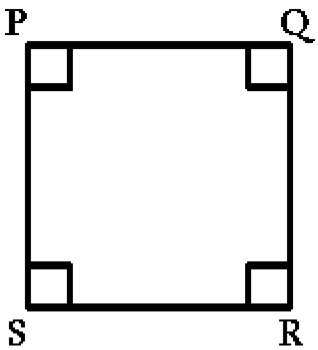
AB ⊥ CD

iii) Line segment: A terminated line is called a line segment. It has two endpoints [A and B in this case.].

iv) Radius of a circle: The distance from the center to any point on the circle is called the radius of the circle.



v) Square: A square is a regular quadrilateral which means that it has four equal sides and four right angles.



Q3. Consider two 'postulates' given below:

(i) Given any two distinct points A and B there exists a third point C which is in between A and B.

(ii) There exist at least three points that are not on the same line.

Do these postulates contain any undefined terms? Are these postulates consistent? Do they follow from Euclid's postulates? Explain.

Answer:

We will use Euclid's axioms here to check these postulates.

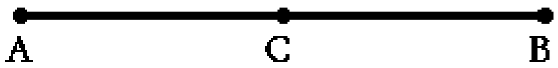
Yes, these postulates contain undefined terms like point and line.

These two statements are consistent as they talk about two different situations meaning different things.

These statements do not follow Euclid's postulates but one of the axioms about "Given any two points, a unique line that passes through them" is followed.

Q4. If a point C lies between two points A and B such that AC = BC, then prove that AC = 1/2 AB. Explain by drawing the figure.

Answer: According to Euclid's axioms, we know that when equals are added to equals, the wholes are equal.



Given: AC = BC

Adding AC on both sides, we get

⇒ AC + AC = BC + AC (BC + AC coincides with AB)

⇒ 2 AC = AB

⇒ AC = 1/2 AB

Q5. In Question 4, point C is called a mid-point of line segment AB. Prove that every line segment has one and only one mid-point.

Answer: We know that the things which coincide with one another are equal to one another.

Let us consider that line segment AB has two midpoints 'C' and 'D' as shown in the figure below.

Let's assume C to be the mid-point of AB

Thus, AC = BC

Adding AC on both sides, we get

⇒ AC + AC = BC + AC (BC + AC coincides to AB)

⇒ 2 AC = AB

⇒ AC = 1/2AB-----(1)

Let us consider a point D lying on AB,

Let's assume that D is another mid-point of AB.

Therefore AD = BD

Adding equal length AD on both sides, we get

AD + AD = BD + AD (BD + AD coincides to AB)

⇒ 2 AD = AB

⇒ AD = 1/2AB------(2)

From equations (1) and (2), we can conclude that AC = AD

- C has to coincide with D for AC to be equal to AD.
- According to Euclid's Axiom 4: Things which coincide with one another are equal to one another.
- Thus, a line segment has only one midpoint.

Q6. In Fig. 5.10, if AC = BD, then prove that AB = CD.

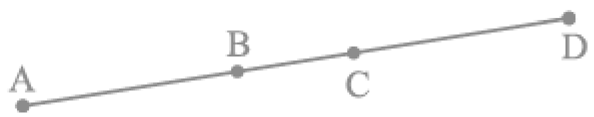


Fig. 5.10

Answer: According to Euclid's axioms, we know that when equals are subtracted from equals, the remainders are equal.

Given: AC = BD

Hence, AB + BC = BC + CD

[Since Point B lies between A and C; Point C lies between B and D]

Subtracting BC from both sides,

⇒ AB + BC - BC = BC + CD - BC

⇒ AB = CD

Q7. Why is Axiom 5, in the list of Euclid's axioms, considered a 'universal truth'? (Note that the question is not about the fifth postulate.)

Answer: Axiom 5 of Euclid's Axioms states that - "The whole is greater than the part."

This axiom is known as a universal truth because it holds true in any field of mathematics and in other disciplinarians of science as well.

- Let us consider a line segment AB. Mark two points P and Q on

AB is a whole part.

It is divided into three parts: AP, PQ, QB.

AB = AP + PQ + QB

Thus, we see that

AB > AP

AB > PQ

AB > QB

Hence, AB (whole) is greater than its parts i.e, AP, PQ, and QB.

Let's take some practical facts to understand this.

- Bangalore is a part of Karnataka which means that Karnataka is larger than Bangalore. i.e. Karnataka > Bangalore.
- India is a part of the world which concludes the world is larger than India. Here the world is a whole whereas, India is just a part of it.

Therefore, it is true that the whole is greater than the part that is considered as universal truth.

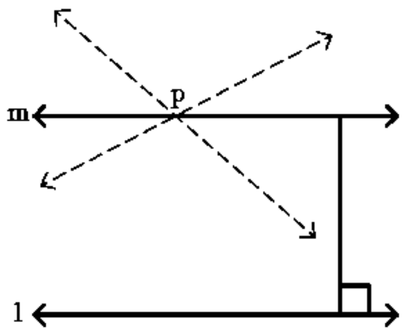
Exercise 5.2

Q1. How would you rewrite Euclid's fifth postulate so that it would be easier to understand?

Answer:

Euclid's fifth postulate: Given a line L and a point P not on the line, exactly one line can be drawn through P which is parallel to L.

For every line 'l' and for every point 'P' not lying on 'l', there exist a unique line 'm' passing through 'P' and parallel to 'l'. This is called 'Playfair's Axiom'

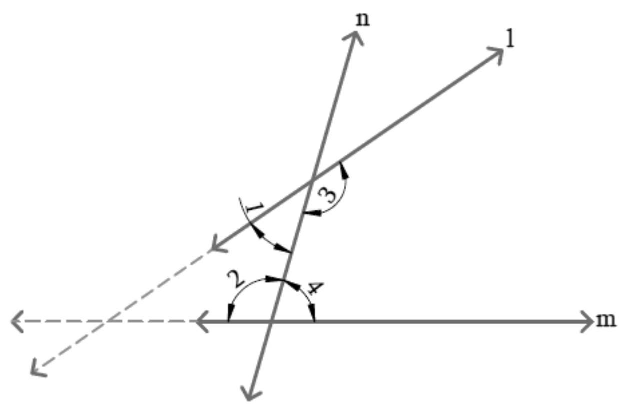


- 'l' is a line and 'p' is a point not lying on 'l'.
- We can draw infinite lines through 'p' but there is only one line unique which is parallel to 'l' and passes through 'p'.
- Take any point on 'l' and draw a line to 'm'. Measure these distances.
- We know that it is the same everywhere, so these lines 'l' and 'm' do not meet anywhere.
- Hence, the two lines 'l' and 'm' are parallel.

Q2. Does Euclid's fifth postulate imply the existence of parallel lines? Explain

Answer: Postulate 5: If a straight line falling on two straight lines makes the interior angles on the same side of it taken together less than two right angles, then the two straight lines, if produced indefinitely, meet on that side on which the sum of angles is less than two right angles.

Yes, if 'a' and 'b' are two straight lines which are intersected by another line 'c', and the sum of co-interior angles are equal to 180°, then a || b.



According to Euclid's 5th postulate,

$\angle 1 + \angle 2 < 180$ then $\angle 3 + \angle 4 > 180$

[The interior angles on the same side of two straight lines which are intersected by another line taken together are less than two right angles]

Producing the line 'a' and 'b' further will meet in the side of which is less than 180°.

If ∠1 + ∠2 = 180 then ∠3 + ∠4 = 180

The lines 'a' and 'b' do not meet in the side where the angle is lesser than 180°

Thus, they will never intersect each other. Hence the two lines are said to be parallel to each other i.e. a || b