Exercise 13.1

Multiple Choice Questions

Q1. In a cylinder, if radius is halved and height is doubled, the volume will be

(A) Same **(B) Doubled** **(C) Halved** **(D) Four times**

Answer: (C) Halved

Radius of cylinder = $\frac{r}{2}$

Height of a cylinder = 2h

Substitute the above values in the volume of a cylinder

Volume of a cylinder = $\pi r^2 h$

Volume of a cylinder = $\pi\left(\frac{r}{2}\right)^2 h$

V = $\pi\left(\frac{r^2}{4}\right)(2h)$

V = $\pi\left(\frac{r^2}{2}\right)h$

V = $\frac{1}{2}\pi r^2 h$

2. The radius of a sphere is 2r, **then its volume will be**

(A) $\frac{4}{3}\pi r^3$ **(B)** $4\pi r^3$ **(C)** $\frac{8}{3}\pi r^3$ **(D)** $\frac{32}{3}\pi r^3$

Answer: (D) $\frac{32}{3}\pi r^3$

Radius of the sphere = 2r

Substitute the value in the formula.

Volume of the sphere = $\frac{4}{3}\pi r^3$

V = $\frac{4}{3}\pi(2r)^3$

V = $\frac{4}{3}\pi(8r)^3$

V = $\frac{32}{3}\pi r^3$

2. The total surface area of a cube is 96 cm^2. The volume of the cube is:

(A) 8 cm^3 (B) 512 cm^3 (C) 64 cm^3 (D) 27 cm^3

Answer: (C) 64 cm^3

Total surface area of a cube = 96 cm^2

Total surface area of cube = $6a^2$

Substitute the total surface area value in total surface area formula.

$6a^2$ = 96 cm

a^2 = 16 cm^2

a = $\sqrt{16}$ cm^2

a = 4 cm^2

Substitute the a value in the volume of the cube formula.

Volume of the cube = a^3

➔ $(4)^3$

➔ 64 cm^3

3. A cone is 8.4cm high and the radius of its base is 2.1 cm. It is melted and recast into a sphere. The radius of the sphere is :

(A) 4.2cm

(B) 2.1cm

(C) 2.4cm

(D) 1.6cm

Answer: (B) 2.1cm

Let r_s be the radius of the sphere

Radius of the cone = 2.1 cm

Height of the cone = 8.4 cm

Volume of the cone = volume of the sphere

$\frac{1}{3}\pi r_c^2 h = \frac{4}{3}\pi r_s^3$

$\frac{1}{3} r_c^2$h = $\frac{4}{3} r_s^3$

r_s^3 = 9.261

r_s = 2.1 cm.

4. In a cylinder, radius is doubled and height is halved, curved surface area will be.

(A) Halved **(B) Doubled** **(C) Same** **(D) Four times**

Answer: (C) Same

Radius of a cylinder = 2r

Height of a cylinder = $\frac{h}{2}$

Substitute above values in curved surface area formula.

Curved surface area of a cylinder = 2πrh

C = 2π (2r) $\left(\frac{h}{2}\right)$

C = 2πrh

The curved surface area will remain the same.

5. The total surface area of a cone whose radius is $\frac{r}{2}$ and slant height 2l is

(A) 2πr(l+r)

(B) πr $\left(l + \frac{r}{4} \right)$

(C) πr(l+r)

(D) 2πrl

Answer: (B) πr $\left(\boldsymbol{l} + \frac{\boldsymbol{r}}{\boldsymbol{4}} \right)$

Radius = $\frac{r}{2}$ slant height = 2l

Therefore, Total surface area of cone = πr (l + r)

= π $\frac{r}{2}$ ($\frac{r}{2}$ + 2l)

= π ($\frac{r^4}{4} + rl$)

= $\pi r\left(l+\frac{r}{4}\right)$

6. The radii of two cylinders are in the ratio of 2:3 and their heights are in the ratio of 5:3 The ratio of their volumes is:

(A) 10:17 (B) 20:27 (C) 17:27 (D) 20:37

Answer: (B) 20:27

Given the radii of two cylinders are in the ratio of 2:3 and their heights are in the ratio of 5:3

Assume the radii r1 and r2 be 2r and 3r respectively and the height h1 and h2 be 5h and 3h respectively.

Therefore, the ratio of their volumes $\frac{V_1}{V_2}=\frac{\pi R_{1h_1}}{\pi R_2 h_2}$

$=\frac{V_1}{V_2}=\frac{4(5)}{9(3)}$

$=\frac{V_1}{V_2}=\frac{20}{27}$

➔ $V_1 : V_2 = 20 : 27$

7. The lateral surface area of a cube is 256m^2. The volume of the cube is

(A) 512m^3 (B) 64m3 6 (C) 216m3 (D) 256m3

Answer: (A) 512m^3

Lateral surface area of a cube =256m2

Lateral surface area of cube = 4a2

Substitute the lateral surface area value in lateral surface area formula.

256m^2 = 4a^2

⇒ a^2 = 64m^2

⇒ a = 8m

Substitute the aa value in the volume of the cube formula.

Volume of the cube

⇒ a^3

⇒ (8)^3

⇒ 512 m^3

8. The number of planks of dimensions (4m × 50cm × 20cm) that can be stored in a pit which is 16m long, 12m wide and 4m deep is

(A) 1900 (B)1920 (C) 1800 (D) 1840

Ans: (B)1920

Dimensions of the plank

l = 4m

b = 50 cm

b = $\frac{50}{100}$ m = 0.5 m

and h = 20 cm

h = 0.2 m

Volume of the plank = l × b × h

Volume of the plank = 4 x 0.5 x 0.3 = 0.4 m^3

Dimensions of the pit are l=16 m, w=12 m and h = 4 m

Volume of the pit = l × b × h

Volume of the pit = 16 x 12 x 4

Number of planks = $\frac{volume\ of\ pit}{volume\ of\ plank}$

= $\frac{16 \text{ x } 12 \text{ x } 4}{0.4}$ = 1920

9. The length of the longest pole that can be put in a room of dimensions (10m × 10m × 5m) **is**

(A) 15m (B) 16m (C) 10m (D) 12m

Answer: (A) 15m

The dimensions are l=10 m ,b = 10 m , h = 5m

Diagonal of cuboid = $\sqrt{l^2 + b^2 + h^2}$

= $\sqrt{100^2 + 10^2 + 5^2}$

= $\sqrt{225}$

= 15 m

10. The radius of a hemispherical balloon increases from 6cm to 12cm as air is being pumped into it. The ratios of the surface areas of the balloon in the two cases is

(A) 1:4 (B) 1:3 (C) 2:3 (D) 2:1

Answer: (A) 1:4

As radius of a hemispherical balloon will be increase from 6 cm to 12 cm.

Assume the radius of a hemispherical balloon r_1 be 6 cm and as air is being pumped into it then radius of a hemispherical balloon r_2 be 12 cm.

$= \frac{r1^2}{r2^2} = \frac{(6)^2}{(12)^2}$

$= \frac{r1^2}{r2^2} = \frac{36}{144}$

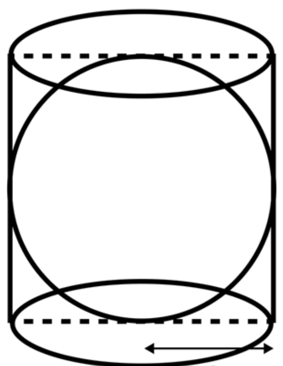
$= \frac{r1^2}{r2^2} = \frac{1}{4}$

➔ $r_1 : r_2 = 1 : 4$

Short Answer Questions with Reasoning

Exercise 13.2

Write True or False and justify your answer.



1. A right circular cylinder just encloses a sphere of radius r as shown in figure. The surface area of the sphere is equal to the curved surface area of the cylinder.

Answer: True

Given, A right circular cylinder just encloses a sphere of radius r

i.e., radius of the sphere == radius of the cylinder =r

and height of the cylinder== diameter of the sphere =2r

Since, Surface area of the sphere=4πr2

Curved surface area of the cylinder =2πrh

Substitute height of the cylinder rh=2r in curved surface area

Curved surface area of the cylinder =2πr(2r) = 4πr2

Therefore, the surface area of the sphere is equal to the curved surface area of the cylinder.

2. An edge of a cube measures r cm. If the largest possible right circular cone is cut out of this cube, then the volume of the cone (in cm3) is $\frac{1}{6}\pi r^3$.

Answer: False

Given, edge of a cube measures r cm

The largest possible right circular cone is cut out of this cube.

Then, diameter of the cone = r cm

radius of the cone = $\frac{r}{2}$

Height of the cone = r

Volume of the cone = $\frac{1}{3}\pi r^3$

$= \frac{1}{3}\pi\left[\frac{r}{2}\right]^2 r$

$= \frac{1}{3}\pi\frac{r^2}{4}r$

$= \frac{1}{12}\pi r^3$

3. The volume of a sphere is equal to two-third of the volume of a cylinder whose height and diameter are equal to the diameter of the sphere.

Answer: True

Assume the radius of the sphere = r

Given the statement, the volume of a sphere is equal to two-third of the volume of a cylinder whose height and diameter are equal to the diameter of the sphere.

Then, the radius of the cylinder = r

And the height of the cylinder =2r

And volume of sphere = $\frac{2}{3}$ volume of cylinder

Since, Volume of the sphere = $\frac{4}{3}\pi r^3$ and volume of the cylinder = $\pi r^2 h$

➔ $\frac{4}{3}\pi r^3 = \frac{2}{3}(\pi r^2(2r))$ because h = 2r

➔ $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi r^3$

Therefore, the volume of a sphere is up to two-thirds of the volume of a cylinder whose height and diameter are up to the diameter of the sphere.

2. If the radius of a right circular cone is halved and height is doubled, the volume will remain unchanged.

Answer: False

Given the statement that the radius of a right circular cone is halved and height is doubled, the volume will remain unchanged is false.

The radius of a right circular cone is halved and height is doubled.

Then, the radius of the circular cone r = $\frac{r}{2}$

Height of the circular cone h = 2h

Substitute the values **r , h** in the formula of volume of cone = $\frac{1}{3}\pi r^2 h$

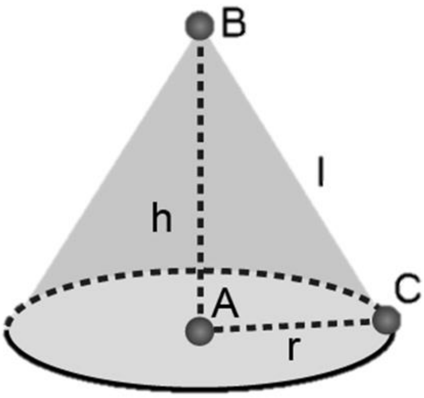
$$V = \frac{1}{3}\pi\left[\frac{r}{2}\right]^2 \times 2h$$

$$V = \frac{1}{3}\pi \times \frac{r^4}{4} \times 2h$$

$$V = \frac{1}{2}\left(\frac{1}{3}\pi r^2 h\right)$$

From the above relation, the radius of a right circular cone is halved and height is doubled, the volume will remain unchanged.

3. In a right circular cone, height, radius, and slant height do not always be sides of a right triangle.



Answer: False

Given statement, In a right circular cone, height, radius, and slant height do not always be sides of a right triangle is true

Consider right circular cone

r is the radius of the cone

h is the height of the cone

l is slant height

By using Pythagoras theorem

$\Rightarrow l^2 = h^2 + r^2$

Therefore, the height, radius, and slant height do not always be sides of a right triangle.

4. If the radius of a cylinder is doubled and its curved surface area is not changed, the height must be halved.

Answer: True

Given, the statement If the radius of a cylinder is doubled and its curved surface area is not changed, the height must be halved is true.

From the given statement,

Let the original dimensions of cylinder,

The radius of a cylinder is r

Height of the cylinder is h.

And the new dimensions of cylinder,

Radius of a cylinder is 2r

The height of the cylinder is h'.

Substitute the radius and height values in the formula curved surface area of a cylinder.

The curved surface area of a cylinder $= 2\pi rh$

$\Rightarrow 2\pi rh = 2\pi(2r) \times h'$

$\Rightarrow h' = \frac{2\pi rh}{4\pi r}$

$\Rightarrow h' = \frac{h}{2}$

Therefore, the height must be halved.

5. The volume of the largest right circular cone can be fitted in a cube whose edge is 2r equals to the volume of a hemisphere of radius r.

Answer: True

Given, the statement the volume of the largest right circular cone that can be fitted in a cube whose edge is 2r equals to the volume of a hemisphere of radius r is true

Given,

Edge of cube is **2r**, then height of the cube **h = 2r.**

The volume of a cone = $\frac{1}{3}\pi r^2 h$

V = $\frac{1}{3}\pi r^2(2r)$

V = $\frac{2}{3}\pi r^3$

Volume of a cone == Volume of a hemisphere of radius r.

6. A cylinder and a right circular cone are having the same base and same height. The volume of the cylinder is three times the volume of the cone.

Answer: True

Given the statement, a cylinder and a right circular cone are having the same base and same height. The volume of the cylinder is three times the volume of the cone is true.

Assume the radius of the base of a cylinder and a right circular cone be rr

and height is h.

Since, Volume of a cylinder = $\pi r^2 h$

Volume of the cone = $\frac{1}{3}\pi r^2 h$

Then,

Volume of a cylinder =3 × Volume of a cone

From the above relation, the volume of the cylinder is 3 times of the volume of a cone.

7. A cone, a hemisphere and a cylinder stand on equal bases and have the same height. The ratio of their volumes is 1:2:3**.**

Answer: True

Given the statement is a cone, a hemisphere and a cylinder stand on equal bases and have the same height. The ratio of their volumes is 1:2:3 is true.

Since, volume of the cylinder is $\pi r^2 h$

Volume of the hemisphere is $\frac{2}{3}\pi r^3$

Volume of the cone is $\frac{1}{3}\pi r^2 h$

The ratio of a cone, hemisphere and a cylinder is $V_e : V_h : V_{eyl}$

➔ $\frac{1}{3}\pi r^2 h : \frac{2}{3}\pi r^3 : \pi r^2 h$

➔ $\frac{1}{3}\pi r^2 : \frac{2}{3}\pi r^3 : \pi r^2$

➔ $\pi r^3 : 2\pi r^3 : 3\pi r^3$

➔ 1: 2 : 3

Therefore, The ratio of a cone, hemisphere and a cylinder is 1:2:3.

8. If the length of the diagonal of a cube is $6\sqrt{3}\ cm$, then the length of the edge of the cube is 3 **cm.**

Answer: False

Given the statement, If the length of the diagonal of a cube is $6\sqrt{3}\ cm$, then the length of the edge of the cube is 3 cm is false.

Given, the length of the diagonal of a cube = $6\sqrt{3}\ cm$

Let the edge of a cube be a cm.

Since, diagonal of a cube = $a\sqrt{3}$

On comparing

$6\sqrt{3}\ cm = a\sqrt{3}$

a = 6 cm.

Therefore, the edge of a cube is 6cm.

9. If a sphere is inscribed in a cube, then the ratio of the volume of the cube to the volume of the sphere will be 6 : π.

Answer: True

Given the statement that if a sphere is inscribed in a cube, then the ratio of the volume of the cube to the volume of the sphere will be **6 : π** is true.

Given a sphere is inscribed in a cube

Calculate the ratio between the volume of a sphere and a volume of a cube

Since, the diameter of the sphere is equal to the side of the cube.

Let the diameter of the sphere be d

Then, radius be $\frac{d}{2}$

Substitute the radius value in volume of a sphere.

Volume of a sphere **(V_s)** = $\left(\frac{4}{3}\right)\pi\left(\frac{d}{2}\right)^3$

$$V_s = \frac{\pi d^3}{6}$$

Volume of a cube $(V_c) = s^3$

$$V_c = d^3$$

Then, the ratio between the volume of a sphere and a volume of a cube = $\frac{V_S}{V_C}$

$$\Rightarrow \frac{V_S}{V_C} = \frac{\left(\frac{\pi d^3}{6}\right)}{d^3}$$

Therefore, the ratio of the volume of the cube to the volume of sphere is 6 : π.

10. If the radius of a cylinder is doubled and height is halved, the volume will be doubled.

Answer: True

Given the statement, if the radius of a cylinder is doubled and height is halved, the volume will be doubled is true.

Let the radius of a cylinder R = 2r

And height of a cylinder, h = $\frac{h}{2}$

Then substitute the values in volume of cylinder V = $\pi r^2 h$

Volume of cylinder V = $\pi\ (2r)^2\ \frac{h}{2}$

V = $2\pi r^2 h$

Therefore, if the radius of a cylinder is doubled and height of a cylinder is halved, then volume of the cylinder is doubled.

Short Answer Questions

1.The surface area of a sphere of radius 5cm is five times the area of the curved surface of a cone of radius 4cm. Find the height and the volume of the cone (taking π = 22/7).

Answer: Given the surface area of a sphere of radius 5 cm5 cm is five times the area of the curved surface of a cone of radius 4 cm.

i.e.., surface area of a sphere = 5 (curved surface of a cone)

$4\pi r^2 = 5(\pi rl)$

Substitute the values of radius in above formula

$\Rightarrow 4 \times 25 = 5 (4 \times l)$

$\Rightarrow 20 = 4l$

$\Rightarrow l = 5cm$

Calculate height of the cone by using the formula $h^2 = l^2 - r^2$

$\Rightarrow h^2 = (5)^2 - (4)^2$

$\Rightarrow h^2 = 25 - 16$

$\Rightarrow h^2 = 9$

$\Rightarrow h = 3$

Now, calculate the volume of the cone by using V = $\frac{1}{3}\pi r^2 h$

➔ $\frac{1}{3} \times \frac{22}{7} \times (4)^2 \times 3$

➔ V = $\frac{352}{7}$

➔V = 50.29 cm^2.

2. The radius of a sphere is increased by 10%10%**. Prove that the volume will be increased by 33.1% approximately.**

Answer: Given the radius of a sphere is increased by 10 %.

10 % increase in radius = 10 % r

⇒ Increased radius = **r +** $\frac{1}{10}$r

⇒ Increased radius = $\frac{11}{10}$r

Substitute the value of increased radius in formula volume of sphere.

Volume of a sphere = $\frac{4}{3}\pi r^3$

V = $\frac{4}{3}\pi\left(\frac{11}{10}r\right)^3$

V = $\frac{4}{3}\pi$ x $\frac{1331}{1000}$ r^3

V = $\frac{4}{3}\pi$ x 1.331x r^3

Difference increased volume and original volume is

= V = $\frac{4}{3}\pi R^3(1.331)$ - $\frac{4}{3}\pi r^3$

= $\frac{4}{3}\pi R^3$ (1.331 – 1)

= $\frac{4}{3}\pi R^3$ (0.331)

The volume will be increased % is

Increase % = $\frac{\frac{4}{3}\pi R^3\ (0.331)}{\frac{4}{3}\pi r^3}$ x 100

Increase % = 33.1 %

Exercise 13.3

1. Metal spheres, each of radius 2cm, are packed into a rectangular box of internal dimensions 16 cm × 8 cm × 8 cm. When 16 spheres are packed the box is filled with preservative liquid. Find the volume of this liquid. Give your answer to the nearest integer. Use π=3.14.

Answer: Given,

Metal sphere of each radius =2cm

And internal dimension of a packed rectangular box is l = 16 cm , b = 8cm h = 8cm

Calculate the volume of a metal sphere

Volume of a sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3}$ x 3.14 x $(2)^3$

$= \frac{4}{3}$ x 3.14 x 8

Now, calculate the volume for 16 metal spheres

Volume of 16 metal spheres = 16 x $\frac{4\ x\ 3.14\ x\ 8}{3}$

$= \frac{100.48\ x\ 16}{3}$

= 535.89 cm^3

Calculate the internal volume of a rectangular box

V = l × b × h

V = 16 x 8 x 8

V = 1024 cm^3

When 16 spheres are packed in the rectangular box is filled with preservative liquid

Calculate the volume of preservative liquid

V_1 = 1024 – 535.9

$V_1 = 488.11$

$V_1 \approx 488\ cm^3$

2. A storage tank is in the form of a cube. When it is full of water, the volume of water is 15.625 m^3.If the present depth of water is 1.3m1.3m**, find the volume of water already used from the tank.**

Answer: Given, a storage tank is in the form of a cube

When the tank is full of water then volume of water is 15.625 m^3

So that Volume of water will be equal to the Volume of cube

Volume of a cube =15.625 m^3

Volume of a cube=a^3

15.625 $m^3 = a^3$

$a = \sqrt[3]{15.625\ m^3}$

a = 2.5 m

If the present depth of the water is 1.3 m i.e., d=2.5 m

Then, Length of tank l = 2.5 m

Breadth of tank b = 2.5 m

Volume of water 1.3 m depth

$V_{1.3}$ = l x b x h

$V_{1.3}$ = 2.5 x 2.5 x 1.3

$V_{1.3}$ = 8.125 m^3

Then, to calculate the volume of water already used from the tank

Volume of water already used from the tank = (Volume of tank when it was full of water) − (Volume of water when depth is 1.3 m)

Volume of water already used from the tank = 15.625 − 8.125

Volume of water already used from the tank = 7.5 m^3

3. Find the amount of water displaced by a solid spherical ball of diameter 4.2 cm4.2 cm**, when it is completely immersed in water**

Answer: Given,

Diameter of a spherical ball is 4.2 cm

Then radius is $\frac{d}{2}$

⇒ r = $\frac{4.2}{2}$

- R = 2.1 cm

When a solid spherical ball is immersed completely in water it will be equal to the volume of the sphere.

Since, volume of the sphere V = $\frac{4}{3}\pi r^3$

- V = $\frac{4}{3}$ X $\frac{22}{7}$ x $(21)^3$

- V = $\frac{814.968}{21}$

V ≈ 38.81 cm^3

4. How many square meters of canvas is required for a conical tent whose height is3.5m3.5m **and the radius of the base is** 12m**?**

Answer: Given,

Height of the conical tent is 3.5 m

Radius of the base is 12 m

Calculate the slant height of conical tent by using the formula l = $\sqrt{h^2 + r^2}$

➔ l = $\sqrt{(3.5)^2 + (12)^2}$

➔ l = $\sqrt{12.25 + 144}$

➔ l = $\sqrt{156.25}$

➔ 12.5 cm

Canvas required for a conical tent is equal to the volume of a cone

Volume of the cone = πrl

$V = \frac{22}{7} \times 12 \times 12.5$

$V = 471.42\ m^2$

5. Two solid spheres made of the same metal have weights 5920 g **and** 740 g**, respectively. Determine the radius of the larger sphere, if the diameter of the smaller one is** 5 cm**.**

Ans: Given, Weights of the two solid spheres w_1 and w_2 are 5920 g and 740 g respectively

Diameter of the smaller solid sphere is d_2 = 5cm

Radius of the smaller solid sphere is $r_2 = \frac{d_2}{2}$

$r_2 = \frac{5}{2}$ cm

Now, calculate the volume of the each sphere by using

Volume = $\frac{Mass}{Density}$

Volume of the larger sphere $V_1 = \frac{5920}{D}$

Volume of the larger sphere $V_2 = \frac{740}{D}$

$\Rightarrow \frac{V_1}{V_2} = \frac{\frac{5920}{D}}{\frac{740}{D}}$

$\Rightarrow \frac{\frac{4}{3}\pi r_1^2}{\frac{4}{3}\pi r_2^2} = \frac{5920}{740}$

$\Rightarrow \left(\frac{r1^2}{\frac{5}{2}}\right)^2 = \frac{592}{74}$

$\Rightarrow \frac{\frac{r^3}{125}}{8} = \frac{592}{74}$

$\Rightarrow \frac{8r^3}{125} = \frac{592}{74}$

➔ $r_1^3 = \frac{592}{74} x \frac{125}{8}$

➔ $r_1^3 = \frac{74000}{592}$

➔ r_1^3 = 125

➔ $r_1 = \sqrt[3]{125}$

➔ r_1 = 5 cm

6. A school provides milk to the students daily in a cylindrical glasses of diameter 7cm. If the glass is filled with milk upto a height of 12cm, find how many litres of milk is needed to serve 1600 students.

Answer: Given , Diameter of cylinder d=7cm Then, Radius of cylinder, r = $\frac{d}{2}$

r = $\frac{7}{2}$

r = 3.5 cm

The glass is filled with milk up to a height, h = 12cm

Then, volume of milk in cylindrical glass is equal to the volume of the cylinder

Volume of the cylinder V = $\pi r^2 h$

➔ V = $\frac{22}{7}$ x 3.5 x 3.5 x 12

➔ V = 462 cm^3

For 1600 students, the volume of the milk is

V= 1600 × 462 cm^3

V = 739200 cm^3

1liter = 1000 cm^3

i.e., $1cm^3$ = 0.001

Then, Volume of milk for 1600 students .

V = 739200 × 0.001

V = 739.2 liters

7. A cylindrical roller 2.5m in length, 1.75m in radius when rolled on a road was found to cover the area of 5500 m^2. How many revolutions did it make?

Answer: Length of cylindrical roller that is equal to height of the cylinder

H = 2.5m

Radius of cylindrical roller, r = 1.75 m

When rolled on a road was found to cover the area of 5500 m^2

Curved surface area of cylindrical roller is equal to the volume of the cylinder

CSA = 2πrh

Substitute the values in the formula of curved surface area

➔ CSA = $2 \times \frac{22}{7} \times 1.75 \times 2.5$

➔ CSA = 27.5 m^2

Area of road covered in one revolution = 27.5 m^2

According to the question, total area of road covered = 5500 m^2

So that, number of revolutions made by road roller to cover 5500 m^2

number of revolutions = $\frac{5500}{27.5}$

number of revolutions = 200

8. A small village, having a population of 5000 requires 75 litres of water per head per day. The village has got an overhead tank of measurement 40m × 25m × 15m. For how many days will the water of this tank last?

Ans: Given, A small village having a population = 5000

Water required per head per day =75L

Determine the volume of water required for a small village per day

v_w =5000 × 75

v_w = 375000L

Since, 1m^3 equal to 1000 L

$v_w = \frac{37500}{1000}$ m^3

v_w = 375 m^3

Total capacity of water in overhead tank will be equal to Volume of overhead tank

➔ v_t = 40 x 25 x 15

➔ v_t =15000 m^3

To find number of days will the water of this tank last is

Number of days = $\frac{V_t}{V_w}$

Number of days = $\frac{15000}{375}$

Number of days = 40 days

9. A shopkeeper has one spherical laddoo of radius 5 cm. With the same amount of material, how many laddoos of radius 2.5 cm can be made?

Ans: Given the spherical laddoo of a radius r = 5cm

Volume of spherical laddoo will be equal to the volume of the sphere

V = $\frac{4}{3}\pi r^3$

$V_5 = \frac{4}{3} \text{x} \frac{22}{7} \text{ x } 5 \text{ x } 5 \text{ x } 5$

$V_5 = 523.81$

With the same amount of material number of laddoos with radius 2.5cm can be made.

Volume of laddoo having the radius 2.5cm.

$V_{2.5} = \frac{4}{3} \text{x} \frac{22}{7} \text{ x } 2.5 \text{ x } 2.5 \text{ x } 2.5$

$V_{2.5}$ = 65.48 cm^3

Therefore, laddoos with radius 2.5cm can be made is

Number of laddoos = $\frac{523.81}{65.48}$

Number of laddoos = 8

10. A right triangle with sides 6 cm, 8 cm and 10 cm is revolved about the side 8 cm. Find the volume and the curved surface of the solid so formed.

Answer: Given, A right triangle with sides 6cm, 8cm and 10cm is revolved about the side 8 cm

Radius of a cone, r= 6 cm

Height of a cone, h = 8cm

Slant height of a cone, l = 10cm

Since, Volume of a cone ,V = $\frac{1}{3}\pi r^2 h$

V = $\frac{1}{3} \times \frac{22}{7} \times 6 \times 6 \times 8$

V = $\frac{6336}{21}$

V = 301.7 cm^3

Now, calculate the curved surface of the area of cone

CSA = πrl

CSA = $\frac{22}{7} \times 6 \times 10$

CSA = $\frac{1320}{7}$

CSA = 188.5 cm^2

Long Answer Question:

1. Rain water which falls on a flat rectangular surface of length 6m and breadth 4m is transferred into a cylindrical vessel of internal radius 20 cm. What will be the height of water in the cylindrical vessel if the rain fall is 1cm. Give your answer to the nearest integer. (Take π=3.14)

Answer: Length of the cuboid = 600cm

Breadth of the cuboid = 400cm

Since, the rainfall is 1cm.

The quantity of Rainwater that falls on a flat rectangular surface will be 600 cm ,400 cm by 1 cm.

The quantity of water that accumulates on a rectangular surface will be the volume of a cuboid

Volume of a cuboid = l x b x h

Substituting values in the volume of the cuboid formula

V= l x b x h

V=600 × 400 × 1

The volume of water on rectangular surface = 240000 cm^3

Since this water on a rectangular surface is transferred to a cylindrical vessel.

Therefore, the quantity of Rainwater that falls on a flat rectangular surface unit gets to be up to the quantity of water entering into the cylindrical vessel.

Assume the height of the cylindrical vessel be h.

Volume of cylinder = $\pi r^2 h$

Substituting values in the volume of a cylinder

$\Rightarrow 240000 = \pi \times (20)^2 \times h$

➔ $h = \frac{240000}{\pi \times 20 \times 20}$

➔ $h = \frac{600}{3.14}$

h = 191cm

Exercise 13.4

1. A cylindrical tube opened at both the ends is made of iron sheet which is 2 cm thick. If the outer diameter is 16 cm and its length is 100 cm , find how many cubic centimetres of iron has been used in making the tube ?

Answer: Length of the cylinder is also equal to the height of the cylinder

So, Height =100cm

Length =100cm

Outer diameter d =16cm

Then, outer radius will be

r=16

r=8cm

Since, volume of outer cylinder = $\pi r^2 h$

$V_0 = \pi(8)^2(100)$

$V_0 = \frac{22}{7} \times 64 \times 100$

V_0 = 20096 cm^2

From the question, cylindrical tube opened at both the ends is made of iron sheet which is 2 cm thick

So that thickness of iron sheet =2cm

Then, inner diameter = outer diameter −2 (thickness of iron sheet) −2 (thickness of iron sheet)

Inner diameter =16 − (2 × 2)

Inner diameter =12cm

Therefore, Inner radius (R) = $\frac{12}{2}$ = 6 cm

Volume of inner cylinder = $\pi R^2 h$

Where

R = inner radius

$V_I = \pi R^2 h$

$V_I = \frac{22}{7} \times (6)^2 \times 100$

V_I = 11304 cm^3

Since, Volume of iron used = Volume of outer cylinder−−Volume of inner cylinder

$V = V_O - V_I$

V = 20096 − 11304

V = 8792 cm^3

2. A semi-circular sheet of metal of diameter 28cm28cm **is bent to form an open conical cup. Find the capacity of the cup.**

Answer: Diameter of the circular sheet =28cm

Radius of the circular sheet = 14cm

Circumference of semi-circular sheet = 14π.

Now the semicircular sheet is bent into an open conical cup.

Slant height of conical cup (l)(l) will be radius of circular sheet

So that, l = 14cm

Circumference of the base of conical cup will be circumference of semicircular sheet

2πR = 14π

R = 7cm

Depth of conical cup will be height of cone

h = $\sqrt{h^2 - r^2}$

h = $\sqrt{(14)^2 - (7)^2}$

h = $\sqrt{196 - 49}$

h = $\sqrt{147}$

h = $7\sqrt{3}$ cm

Capacity of conical cup will be Volume of cone

V = $\frac{1}{3}\pi R^2 h$

V = $\frac{1}{3}$ x $\frac{22}{7}$ x 7^2 x $7\sqrt{3}$ cm

V = $\frac{1078\sqrt{3}}{3}$ cm^3

3. A cloth having an area of 165 m^2 is shaped into the form of a conical tent of radius 5m

(i) How many students can sit in the tent if a student, on an average, occupies 5/7 m^2 on the ground?

Answer: Curved Surface Area of the cone is165 m^2

Radius of tent is 5m

Space occupied by each candidate is57 m^2

Calculate the number of candidates can sit on the ground

Number of candidates can sit on the ground = $\frac{\text{Curved Surface Area of the base}}{\text{space occupied by each candidate}}$

$= \pi r^2 \div \frac{5}{7}$

= $\frac{22}{7}$ x 25 x $\frac{7}{5}$

= 110

(ii) Find the volume of the cone.

Answer: To find the volume of the cone first calculate the slant height of the cone.

Since, CSA of cone = 165 m2

Radius of tent is 5m

Curved Surface Area = πrl

l is slant height

Slant height = $\sqrt{r^2 + h^2}$

165 = $\frac{22}{7}$ x 5 x $\sqrt{25 + h^2}$

10.5 = $h^2 + 25$

Squaring on both the sides

$h^2 = 110.25 - 25$

$h^2 = 85.23$

h = 9.23 m

Volume of the cone = $\frac{1}{3}\pi r^2 h$

V = $\frac{1}{3}$ x $\frac{22}{7}$ x 25 x 9.23

V = 241.74 m^3

4. The water for a factory is stored in a hemispherical tank whose internal diameter is 14m. The tank contains 50 kilolitres of water. Water is pumped into the tank to fill its capacity. Calculate the volume of water pumped into the tank.

Answer: Internal diameter = 14m

Thus, radius r = $\frac{14}{2}$

➔ r = 7 cm

Capacity of the tank will be volume of the hemisphere

Volume of the hemisphere = $\frac{2}{3}\pi r^3$

V = $\frac{2}{3}$ x $\frac{22}{7}$ x $(7)^3$

V = $\frac{2}{3}$ x $\frac{22}{7}$ x 343

V = 718.67 cm^3

We know that 50 kilolitres = 50 m^3

Volume of water pumped into the tank == Capacity of the tank −50

=718.67 − 50

= 668.66 m^3

5. The volumes of the two spheres are in the ratio 64:27**. Find the ratio of their surface areas.**

Answer: Given, the ratio of volume of the two spheres are 64:27

Assume the radius of two spheres are r_1 and r_2 respectively.

Therefore,

Volume of the sphere of radius r_1

$V_1 = \frac{4}{3}\pi r_1^3$

Volume of the sphere of radius r_2

$V_2 = \frac{4}{3}\pi r_2^3$

Calculate the radius by using the ratio of volume of the two spheres

$V_1 : V_2 = \frac{64}{27}$

$V_1 : V_2 = \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3}$

➔ $\frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{64}{27}$

➔ $\frac{r_1^3}{r_2^3} = \frac{64}{27}$

➔ $\frac{r^1}{r^2} = \frac{4}{3}$

➔ $r_1 : r_2 = 4 : 3$

From the radius calculate the ratios of the surface area of a sphere

Now, ratios of surface area $\frac{S_1}{S_2} = \left(\frac{r^1}{r^2}\right)^2$

$\frac{S_1}{S_2} = \left(\frac{4}{3}\right)^2$

$\frac{S_1}{S_2} = \frac{16}{9}$

$\Rightarrow s_1 : s_2 = 16 : 9$

6. A cube of side 4cm4cm **contains a sphere touching its sides. Find the volume of the gap in between.**

Ans: Given, Side of a cube (a) = 4cm

Calculate the volume of the cube

Volume of cube is $V = a^3$

$V = (4cm)^3$

$V = 64\ cm^3$

Diameter of the sphere will be equal to length of the side of the cube

Side of the cube = 4 cm

Diameter of the sphere = 4 cm

Then, Radius of sphere = 2 cm

Volume of the sphere = $\frac{4}{3}\pi r^3$

$V = \frac{4}{3} \times \frac{22}{7} \times (2)^3$

$V = 33{\cdot}52\ cm^3$

Therefore, Volume of gap = Volume of a cube − Volume of sphere

$V = 64cm^2 - 33{\cdot}52\ cm^3$

$V = 30{\cdot}48\ cm^3$

7. A sphere and a right circular cylinder of the same radius have equal volumes. By what percentage does the diameter of the cylinder exceed its height?

Answer: Assume that radius of sphere and radius of a right circular cylinder is r and height of the cylinder be h.

From the given question, sphere and a right circular cylinder of the same radius have equal volumes

Since, Volume of cylinder = volume of a sphere

Volume of sphere = $\frac{4}{3}\pi r^3$

Volume of cylinder = $\pi r^2 h$

$\pi r^2 h = \frac{4}{3}\pi r^3$

h = $\frac{4}{3}r$

Since, Diameter of the cylinder d = 2r

h = $\frac{2(2r)}{3}$

h = $\frac{2d}{3}$

3h = 2d

d = $\frac{3h}{2}$

The difference between the length of diameter and height is d − h

= $\frac{3h}{2} - h = \frac{h}{2}$

The percentage the diameter of the cylinder exceed its height

= $\frac{\frac{h}{2}}{h}$ x 100 = $\frac{100}{2}$ = 50 %

8. 30 circular plates, each of radius 14cm and thickness 3cm are placed one above the another to form a cylindrical solid. Find :

i) the total surface area

Ans: Given, Radius of the base of the cylinder formed (r)=14cm

Thickness = 3 cm

Height of the cylinder formed (h) = 30 × 3 = 90cm

Calculate the total surface area of the cylinder

Total surface area of the cylinder =2πr(r + h)

= 2 x $\frac{22}{7}$ x 14 (14 + 90)

= 2 x $\frac{22}{7}$ x 14 x 104

= 9152 cm^2

ii) volume of the cylinder so formed.

Answer: Calculate the volume of the cylinder

Volume of the cylinder formed = $\pi r^2 h$

V = $\frac{22}{7}$ x 14 x 14 x 90

V = 55440 cm^3