Exercise 6.1

Write the correct answer in each of the following:

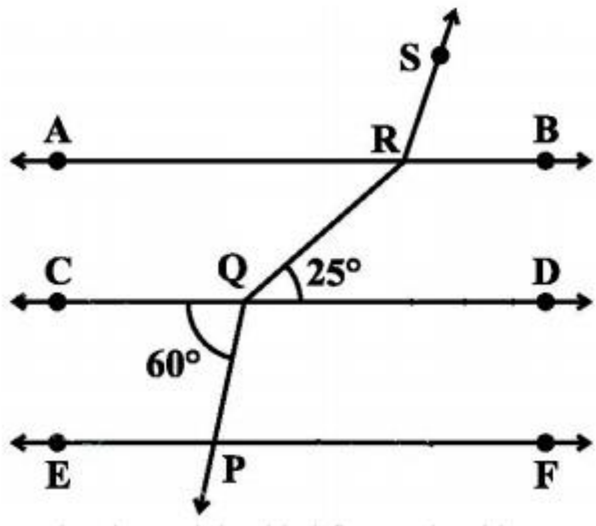
1. In Fig. 6.1, if AB || CD || EF, PQ || RS, ∠RQD = 25° and ∠CQP = 60°, then ∠QRS is equal to

(A) 85°

(B) 135°

(C) 145°

(D) 110°



Answer:

(C) 145°

Explanation:

According to the given figure, we have

AB || CD || EF

PQ || RS

∠RQD = 25°

∠CQP = 60°

PQ || RS.

We know that,

If a transversal intersects two parallel lines, then each pair of alternate exterior angles is equal.

Now, since PQ || RS

⇒ ∠PQC = ∠BRS

We have ∠PQC = 60°

⇒ ∠BRS = 60° … eq.(i)

We also know that,

If a transversal intersects two parallel lines, then each pair of alternate interior angles is equal.

Now again, since AB || CD

⇒ ∠DQR = ∠QRA

We have ∠DQR = 25°

⇒ ∠QRA = 25° … eq.(ii)

Using linear pair axiom,

We get,

∠ARS + ∠BRS = 180°

⇒ ∠ARS = 180° – ∠BRS

⇒ ∠ARS = 180° – 60° (From (i), ∠BRS = 60°)

⇒ ∠ARS = 120° … eq.(iii)

Now, ∠QRS = ∠QRA + ∠ARS

From equations (ii) and (iii), we have,

∠QRA = 25° and ∠ARS = 120°

Hence, the above equation can be written as:

∠QRS = 25° + 120°

⇒ ∠QRS = 145°

Therefore, option (C) is the correct answer.

2. If one angle of a triangle is equal to the sum of the other two angles, then the triangle is

(A) An isosceles triangle

(B) An obtuse triangle

(C) An equilateral triangle

(D) A right triangle

Answer:

(D) A right triangle

Explanation:

Let the angles of △ABC be ∠A, ∠B, and ∠C

Given that ∠A= ∠B+∠C …(eq1)

But, in any △ABC,

Using the angle sum property, we have,

∠A+∠B+∠C=180^{o} …(eq2)

From equations (eq1) and (eq2), we get

∠A+∠A=180^{o}

⇒2∠A=180^{o}

⇒∠A=180^{o}/2 = 90^{o}

⇒∠A = 90^{o}

Hence, we get that the triangle is a right triangle

Therefore, option (D) is the correct answer.

3. An exterior angle of a triangle is 105°, and its two interior opposite angles are equal. Each of these equal angles is

(A) 37 ½^{o}

(B) 52 ½^{o}

(C) 72 ½^{o}

(D) 75°

Solution:

(B) 52 ½^{o}

Explanation:

According to the question,

The exterior angle of triangle = 105°

Let the two interior opposite angles of the triangle = x

We know that,

The exterior angle of a triangle = sum of interior opposite angles

Then, we have the equation,

105° = x + x

2x = 105°

x = 52.5°

x = 52½

Therefore, option (B) is the correct answer.

4. The angles of a triangle are in the ratio 5 : 3 : 7. The triangle is

(A) An acute angled triangle

(B) An obtuse-angled triangle

(C) A right triangle

(D) An isosceles triangle

Solution:

(A) An acute angled triangle

Explanation:

According to the question,

The angles of a triangle are of the ratio 5 : 3 : 7

Let 5:3:7 be 5x, 3x and 7x

Using the angle sum property of a triangle,

5x + 3x +7x =180

15x=180

x=12

Substituting the value of x, x = 12, in 5x, 3x and 7x we get,

5x = 5×12 = 60°

3x = 3×12 = 36°

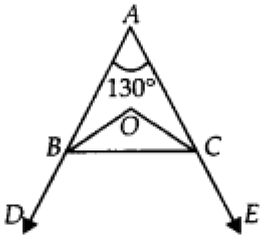
7x = 7×12 = 84°

Since all the angles are less than 90°, the triangle is acute-angled.

Therefore, option (A) is the correct answer.

Q5. If one of the angles of a triangle is 130°, then the angle between the bisectors of the other two angles can be
(A) 50°
(B) 65°
(C) 145°
(D) 155°
Answer:

(D) : Let angles of a triangle be ∠A, ∠B and ∠C, where ∠A = 130°



In ΔABC,

$\angle A + \angle B + \angle C = 180°$

[Angle sum property of a triangle]

$\Rightarrow \frac{1}{2}\angle A + \frac{1}{2}\angle B + \frac{1}{2}\angle C = \frac{180°}{2} = 90°$

[On dividing both sides by 2]

$\Rightarrow \frac{1}{2}\angle B + \frac{1}{2}\angle C = 90° - \frac{1}{2}\angle A$

$\Rightarrow \frac{1}{2}\angle B + \frac{1}{2}\angle C = 90° - \frac{130°}{2}$

$\Rightarrow \frac{1}{2}\angle B + \frac{1}{2}\angle C = 25°$ ...(i)

Now, in ΔOBC

$\angle OBC + \angle OCB + \angle BOC = 180°$

$\Rightarrow \frac{1}{2}\angle B + \frac{1}{2}\angle C + \angle BOC = 180°$ ...(ii)

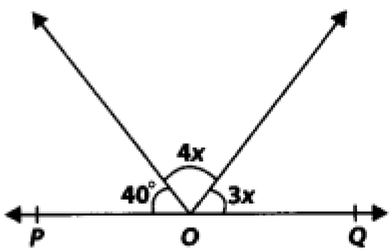
[∵ BO and CO are bisectors of ∠B and ∠C respectively.]

From (i) and (ii), we get

$\angle BOC = 180° - 25° = 155°$

Thus, the required angle is 155°.

Q6. In the given figure, POQ is a line. The value of x is



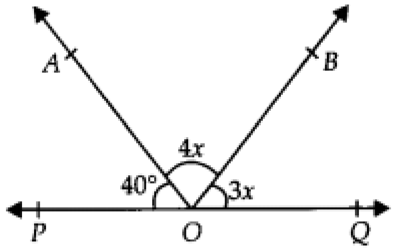
(A) 20°

(B) 25°

(C) 30°

(D) 35°

Answer: (A)) 20°



Since, POQ is a line segment.

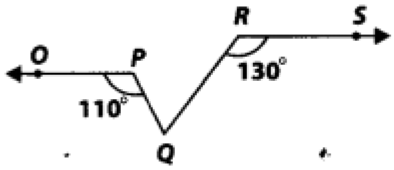
∴ ∠POQ = 180°

⇒ ∠POA + ∠AOB + ∠QOB = 180°

⇒ 40° + 4x + 3x = 180°

⇒ 7x = 180° – 40° ⇒ 7x = 140° ⇒ x = 20°

Q7. In the given figure, if OP || RS, ∠OPQ = 110° and ∠QRS = 130°, then ∠PQR is equal to



(A) 40°

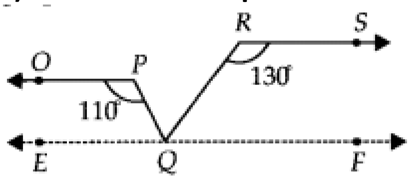
(B) 50°

(C) 60°

(D) 70°

Answer:

(C) : Draw a line EF parallel to RS through point Q



∵ OP || RS [Given]

⇒ EF || RS [Construction]

∴ OP || EF and PQ is a transversal

⇒ ∠OPQ = ∠PQF [Alternate interior angles]

⇒ ∠PQF = 110° [∵ ∠OPQ = 110°]

⇒ ∠PQR + ∠RQF = 110° … (i)

Now, RS || EF and RQ is a transversal

⇒ ∠QRS + ∠RQF = 180° [Co-interior angles]

⇒ 130° + ∠RQF = 180°

⇒ ∠RQF = 180° – 130° = 50°

Now from (i), we have

⇒ ∠PQR + 50° = 110°

⇒ ∠PQR = 110° – 50°

⇒ ∠PQR = 60°

Q8. Angles of a triangle are in the ratio 2 : 4 : 3. The smallest angle of the triangle is
(A) 60°
(B) 40°
(C) 80°
(D) 20°
Answer: (B) 40°

We have given, the ratio of angles of a triangle is 2 : 4 : 3.

Let the angles of a triangle be ∠A, ∠B and ∠C, where ∠A = 2x, ∠B = 4x and ∠C = 3x

Now in Δ ABC, ∠A + ∠B + ∠C = 180°

[Angle sum property of a triangle]

⇒ 2x + 4x + 3x = 180°

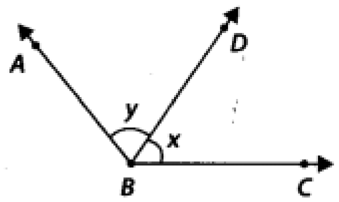
⇒ 9x = 180° ⇒ x = 20°

∠A = 2 × 20° = 40°, ∠B = 4 × 20° = 80° and ∠C = 3 × 20° = 60°

Thus, the smallest angle of the triangle is 40°.

Exercise 6.2

Q1. For what value of x + y in the given figure will ABC be a line? Justify your answer.



Answer: For ABC to be a line, the sum of the two adjacent angles must be 180° i.e., x + y = 180°.

Q2. Can a triangle have all angles less than 60°? Give a reason for your answer.

Answer: No. A triangle cannot have all angles less than 60°

Justification:

According to the angle sum property,

We know that the sum of all the interior angles of a triangle should be = 180°.

Suppose, all the angles are 60°,

Then we get, 60° + 60° + 60° = 180°.

Now, considering angles less than 60°,

Let us take 59°, which is the highest natural number less than 60°.

Then we have,

$59^o + 59^o + 59^o = 177^o \neq 180^o$

Hence, we can say that if all the angles are less than 60°, the measure of the angles won't satisfy the angle sum property.

Therefore, a triangle cannot have all angles less than 60°.

Q3. Can a triangle have two obtuse angles? Give a reason for your answer.

Answer: No. A triangle cannot have two obtuse angles

Justification:

According to the angle sum property,

We know that the sum of all the interior angles of a triangle should be = 180°.

An obtuse angle is one whose value is greater than 90° but less than 180°.

Considering two angles to be equal to the lowest natural number greater than 90°, i.e., 91°.

According to the question,

If the triangle has two obtuse angles, then there are two angles which are at least 91° each.

On adding these two angles,

Sum of the two angles = 91° + 91°

⇒ Sum of the two angles = 182°

The sum of these two angles already exceeds the sum of three angles of the triangle, even without considering the third angle.

Therefore, a triangle cannot have two obtuse angles.

Q4. How many triangles can be drawn having angles as 45°, 64°, and 72°? Give a reason for your answer.

Answer: No triangle can be drawn having its angles 45°, 64° and 72°.

Justification:

According to the angle sum property,

We know that the sum of all the interior angles of a triangle should be = 180°.

But, according to the question,

We have the angles 45°, 64° and 72°.

Sum of these angles = 45° + 64° + 72°

= 181°, which is greater than 180°.

Hence, the angles do not satisfy the angle sum property of a triangle.

Therefore, no triangle can be drawn having its angles 45°, 64° and 72°.

Q5. How many triangles can be drawn having their angles as 53°, 64° and 63°? Give a reason for your answer.

Answer: Infinitely many triangles can be drawn having its angles as 53°, 64° and 63°.

Justification:

According to the angle sum property,

We know that the sum of all the interior angles of a triangle should be = 180°.

According to the question,

We have the angles 53°, 64°, and 63°.

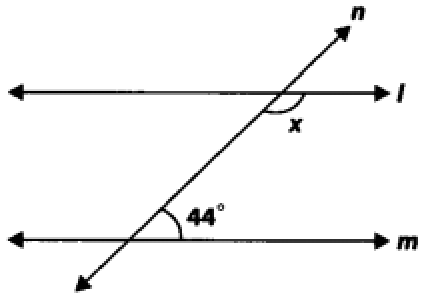
Sum of these angles = 53° + 64° + 63°

= 180°

Hence, the angles satisfy the angle sum property of a triangle.

Therefore, infinitely many triangles can be drawn having its angles as 53°, 64° and 63°.

Q6. In the given figure, find the value of x for which the lines l and m are parallel.



Answer: We have given, l || m and a transversal line n,
∴ x + 44° = 180° [Co-interior angles]
⇒ x = 180° – 44° ⇒ x = 136°

Q7. Two adjacent angles are equal. Is it necessary that each of these angles will lie at a right angle? Justify your answer.

Answer: No, because each of the two adjacent angles will be right angles only if they will form a linear pair.

Q8. If one of the angles formed by two intersecting lines is a right angle, what can you say about the other three angles? Give a reason for your answer.

Answer:

Let two lines AB and CD intersect each other at a right angle.

Let $\angle AOC = 90°$

$\angle AOC + \angle AOD = 180°$ [Linear pair]

$\Rightarrow \angle AOD = 180° - 90° = 90°$

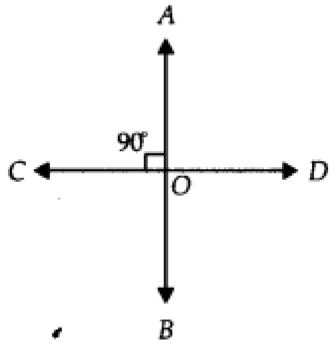
Now, $\angle COA = \angle DOB = 90°$

[Vertically opposite angles]

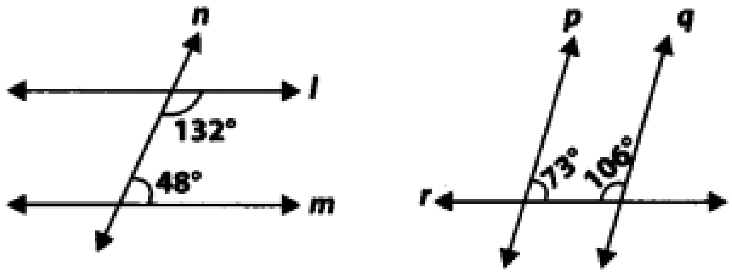
and $\angle AOD = \angle COB = 90°$

[Vertically opposite angles]

Hence, each of the other three angles are right angles.



Q9. In the given figure, which of the two lines are parallel and why?



Answer:

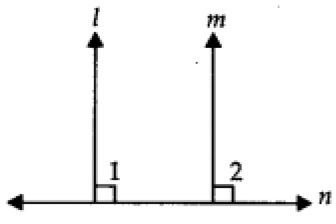
Left side figure shows the sum of two interior angles = 132° + 48° = 180° because the sum of two interior angles on the same side of a transversal line n is 180°, thus l | |m.

Right side figure shows the sum of two interior angles = 73° + 106° = 179° ≠ 180° because the sum of two interior angles on the same side of a transversal line r is not equal to 180°, thus p is not parallel to q.

Q10. Two lines l and m are perpendicular to the same line n. Are l and m perpendicular to each other? Give a reason for your answer.

Solution:

No



Given that the lines l and m are perpendicular to the line n.

∴ ∠1 = ∠2 = 90°

This shows that the corresponding angles are equal.

Thus, l || m.

Exercise 6.3

Q1. In Fig. 6.9, OD is the bisector of ∠AOC, OE is the bisector of ∠BOC and OD ⊥ OE. Show that points A, O and B are collinear.

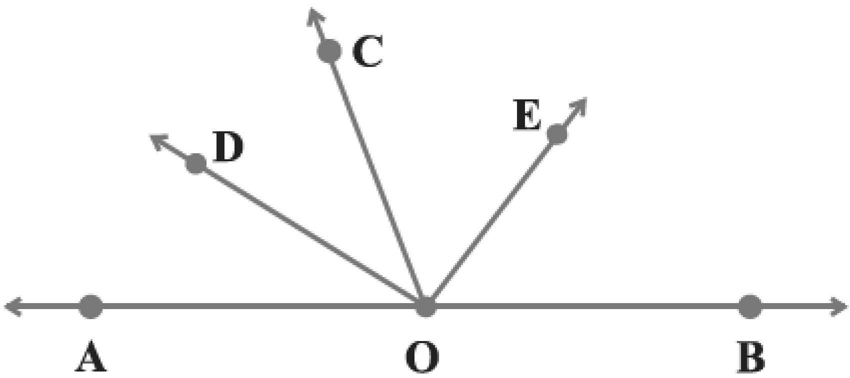


Fig. 6.9

Solution:

According to the question,

In figure,

OD ⊥ OE,

OD and OE are the bisectors of ∠AOC and ∠BOC.

To prove: Points A, O and B are collinear

i.e., AOB is a straight line.

Proof:

Since OD and OE bisect angles ∠AOC and ∠BOC, respectively.

∠AOC = 2∠DOC …(eq.1)

And ∠COB = 2∠COE …(eq.2)

Adding (eq.1) and (eq.2), we get

∠AOC = ∠COB = 2∠DOC + 2∠COE

∠AOC +∠COB = 2(∠DOC +∠COE)

∠AOC + ∠COB = 2∠DOE

Since, OD⊥OE

We get,

∠AOC +∠COB = 2×90°

∠AOC +∠COB =180°

∠AOB =180°

So, ∠AOC + ∠COB form linear pair.

Therefore, AOB is a straight line.

Hence, points A, O and B are collinear.

Q2. In Fig. 6.10, ∠1 = 60° and ∠6 = 120°. Show that the lines m and n are parallel.

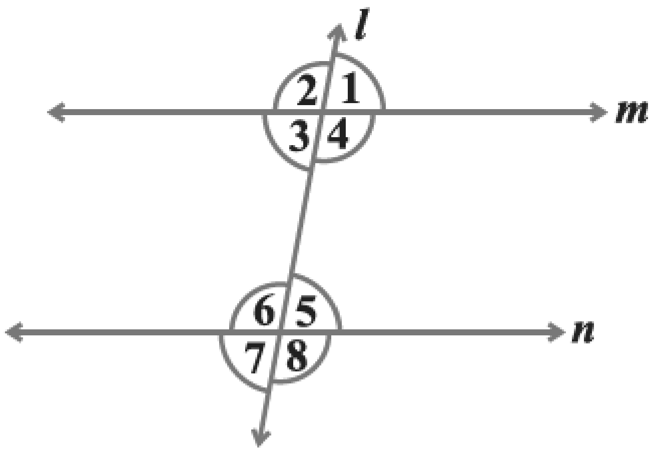


Fig. 6.10

Answer:

According to the question,

We have from the figure ∠1 = 60° and ∠6 = 120°

Since, ∠1 = 60° and ∠6 = 120°

Here, ∠1 = ∠3 [since they are vertically opposite angles]

∠3 = ∠1 = 60°

Now, ∠3 + ∠6 = 60° + 120°

⇒ ∠3 + ∠6 = 180°

We know that,

If the sum of two interior angles on the same side of *l* is 180°, then the lines are parallel.

Therefore, m || n

Q3. AP and BQ are the bisectors of the two alternate interior angles formed by the intersection of a transversal t with parallel lines l and m (Fig. 6.11). Show that AP || BQ.

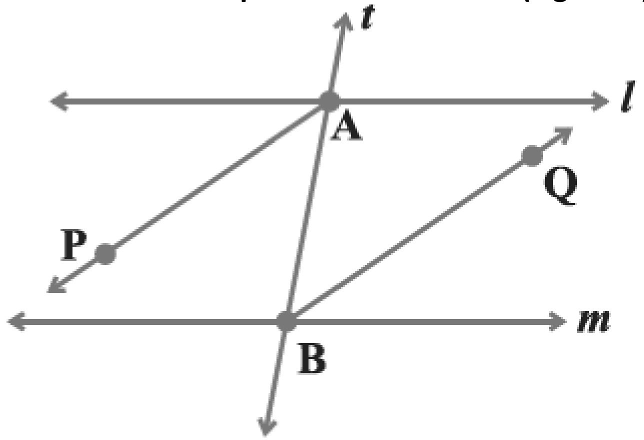
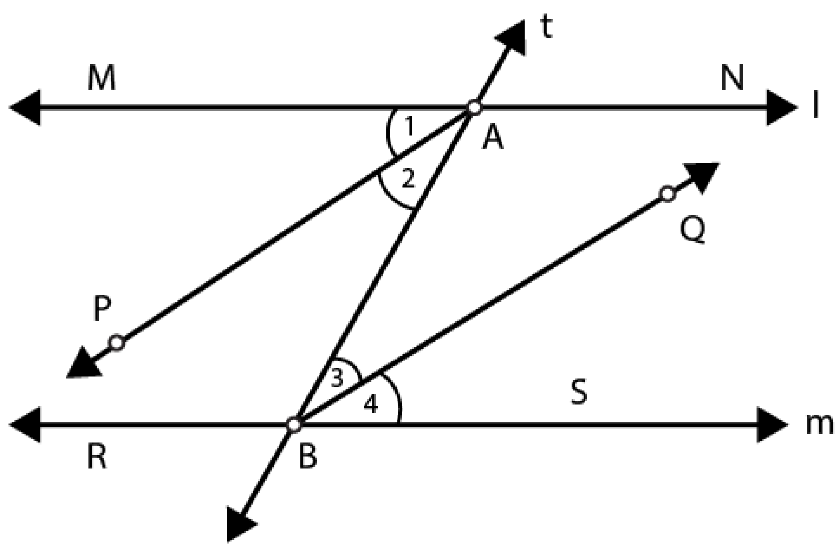


Fig. 6.11

Answer:



l || m and t is the transversal

∠MAB = ∠SBA [alternate angles]

⇒ ½ ∠MAB = ½ ∠SBA

⇒ ∠PAB = ∠QBA

⇒ ∠2 = ∠3

But, ∠2 and ∠3 are alternate angles.

Hence, AP||BQ.

A4. If in Fig. 6.11, bisectors AP and BQ of the alternate interior angles are parallel, then show that *l* || m.

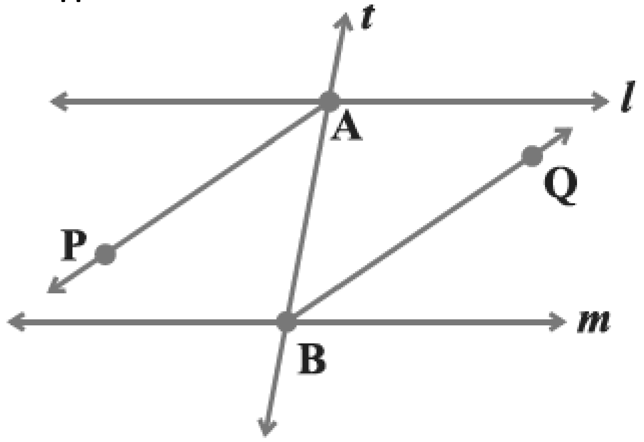


Fig. 6.11

Answer:

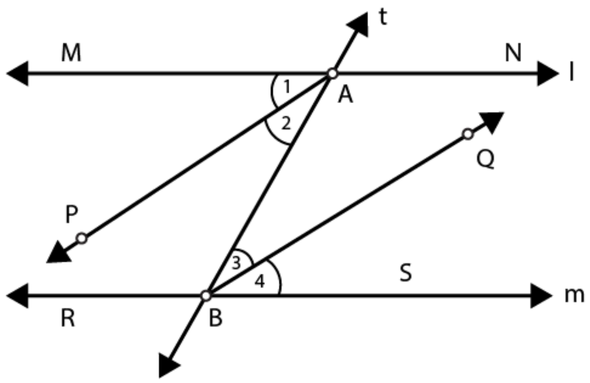
AP is the bisector of ∠MAB

BQ is the bisector of ∠SBA.

Given: AP||BQ.

As AP||BQ,

We have,



So ∠2 = ∠3 [Alternate angles]

2∠2 = 2∠3

⇒ ∠2 + ∠2 = ∠3 +∠3

From figure, we have ∠1= ∠2and ∠3 = ∠4

⇒ ∠1+ ∠2 = ∠3 +∠4

⇒ ∠MAB = ∠SBA

But, we know that these are alternate angles.

Hence, the lines *l* and m are parallel, i.e., l||m.

Q5. In Fig. 6.12, BA || ED and BC || EF. Show that ∠ABC = ∠DEF [Hint: Produce DE to intersect BC at P (say)].

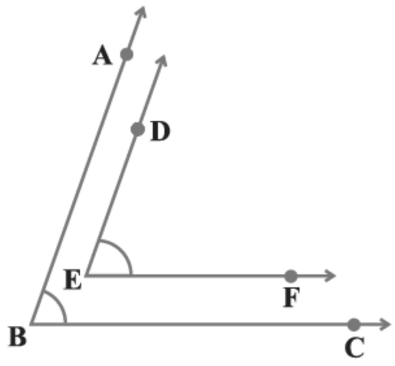


Fig. 6.12

Answer:

Construction:

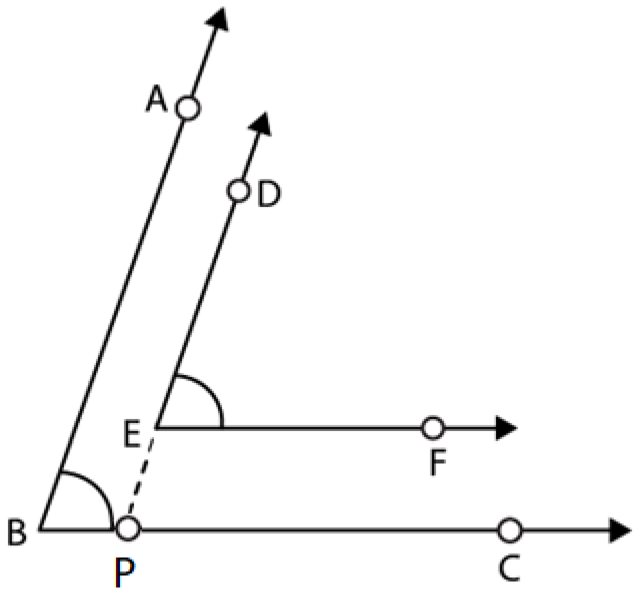
Extend DE to intersect BC at point, P.

Given, EF||BC and DP are the transversal,

∠DEF = *∠DPC* …(eq.1) [Corresponding angles]

Also given, AB||DP and BC is the transversal,

∠DPC = *∠ABC* …(eq.2) [Corresponding angles]

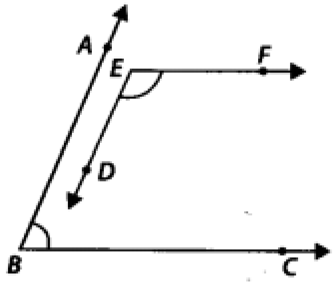


From (eq.1) and (eq.2), we get

∠ABC = ∠DEF

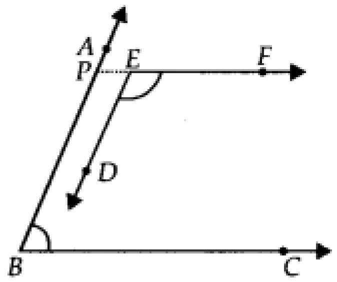
Hence, Proved.

Q6. In the given figure, BA || ED and BC || EF. Show that ∠ABC + ∠DEF = 180°.



Answer:

Let us produce FE, which meets AB at P.



BC || EF ⇒ BC || PF

∴ ∠EPB + ∠PBC = 180° …(i)

[Co-interior angles]

Now, AB || ED and PF is a transversal.

∴ ∠EPB = ∠DEF …(ii)

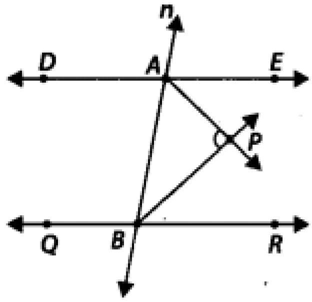
[Corresponding angles]

From (i) and (ii), we get

∠DEF + ∠PBC = 180°

⇒ ∠ABC + ∠DEF = 180° [∵ ∠PBC = ∠ABC]

Q7. In the given figure, DE || QR and AP, and BP are bisectors of ∠EAB and ∠RBA, respectively. Find ∠APB.



Answer:

DE || QR and AB is a transversal

∴ ∠EAB + ∠RBA = 180° (Co-interior angles)

$\Rightarrow \frac{1}{2}\angle EAB + \frac{1}{2}\angle RBA = 90°$ ...(i)

[On dividing both sides by 2]

∵ AP and BP are the bisectors of ∠EAB and ∠RBA, respectively.

$\therefore \angle BAP = \frac{1}{2}\angle EAB$ and $\angle ABP = \frac{1}{2}\angle RBA$

Using these in (i), we have

⇒ ∠BAP + ∠ABP = 90° ...(ii)

In ΔAPB, ∠BAP + ∠ABP + ∠APB = 180°

[Angle sum property of a triangle]

⇒ 90° + ∠APB = 180° [From (ii)]

⇒ ∠APB = 90°

Q8. The angles of a triangle are in the ratio 2 : 3: 4. Find the angles of the triangle.

Answer:

Let the angles of a triangle be 2x, 3x and 4x. Since the sum of all angles of a triangle is 180°.

∴ 2x + 3x + 4x = 180°

$\Rightarrow x = \frac{180°}{9} = 20°$

∴ The required three angles are 2 × 20° = 40°, 3 × 20° = 60° and 4 × 20° = 80°.

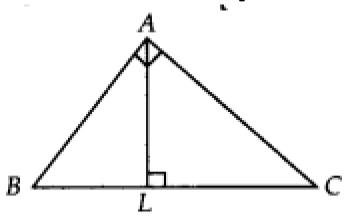
Q9. A triangle ABC is right-angled at A. L is a point on BC such that AL ⊥ BC. Prove that ∠BAL = ∠ACB.

Answer:

In ΔABC and ΔALB,

∠BAC = ∠ALB [Each 90°]…(i)

and ∠ABC = ∠ABL [Common angle]…(ii)



On adding (i) and (ii), we get

∠BAC + ∠ABC = ∠ALB + ∠ABL …(iii)

Again, in ΔABC,

∠BAC + ∠ACB + ∠ABC = 180°

[Angle sum property of a triangle]

= ∠BAC + ∠ABC = 180° – ∠ACB …(iv)

In ΔABE,

∠ABL + ∠ALB + ∠BAL = 180°

[Angle sum property of a triangle]

⇒ ∠ABL + ∠ALB = 180° – ∠BAL …(v)

On substituting the values from (iv) and (v) in (iii), we get

180° – ∠ACB = 180° – ∠BAL

⇒ ∠ACB = ∠BAL

Q10.Two lines are respectively perpendicular to two parallel lines. Show that they are parallel to each other.

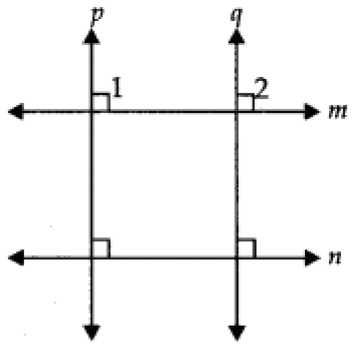
Answer:

Let two lines m and n are parallel and p and q are respectively perpendicular to m and n.

Since, p ⊥ m ⇒ ∠1 = 90° …(i)

Also, q ⊥ m ⇒ ∠2 = 90° …(ii)

[Since, m || n and q ⊥ n ⇒ q ⊥ m]



From (i) and (ii), we have

$Z_1 = Z_2 = 90°$

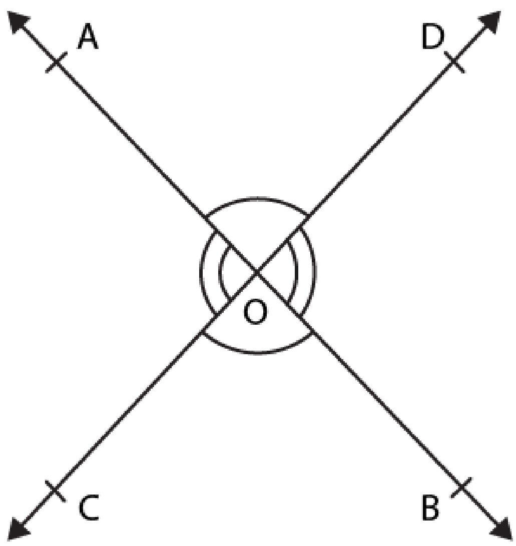
As p and q are two lines and m is transversal. Also, corresponding angles ∠1 and ∠2 are equal.

Thus, p || q.

Exercise 6.4

Q1. If two lines intersect, prove that the vertically opposite angles are equal.

Answer:



From the figure, we know that,

AB and CD intersect each other at point O.

Let the two pairs of vertically opposite angles be,

1st pair – ∠AOC and ∠BOD

2nd pair – ∠AOD and ∠BOC

To prove:

Vertically opposite angles are equal,

i.e., ∠AOC = ∠BOD, and ∠AOD = ∠BOC

From the figure,

The ray AO stands on the line CD.

We know that,

If a ray lies on a line, then the sum of the adjacent angles is equal to 180°.

⇒ ∠AOC + ∠AOD = 180° (By linear pair axiom) … (i)

Similarly, the ray DO lies on line AOB.

⇒ ∠AOD + ∠BOD = 180° (By linear pair axiom) … (ii)

From equations (i) and (ii),

We have,

∠AOC + ∠AOD = ∠AOD + ∠BOD

⇒ ∠AOC = ∠BOD – – – – (iii)

Similarly, the ray BO lies on the line COD.

⇒ ∠DOB + ∠COB = 180° (By linear pair axiom) – – – – (iv)

Also, the ray CO lies on line AOB.

⇒ ∠COB + ∠AOC = 180° (By linear pair axiom) – – – – (v)

From equations (iv) and (v),

We have,

∠DOB + ∠COB = ∠COB + ∠AOC

⇒ ∠DOB = ∠AOC – – – – (vi)

Thus, from equation (iii) and equation (vi),

We have,

∠AOC = ∠BOD, and ∠DOB = ∠AOC

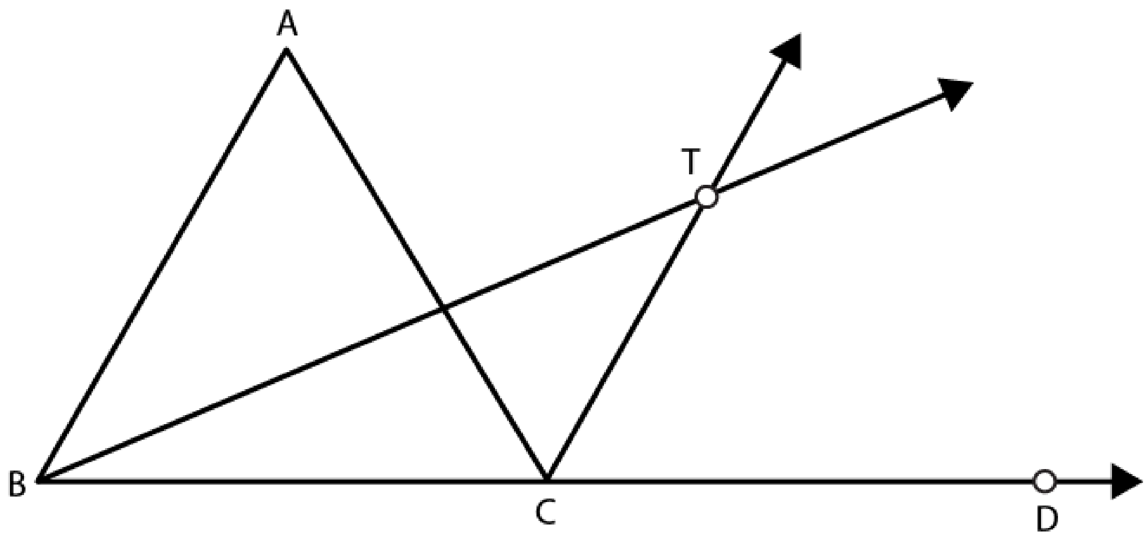
Therefore, we get vertically opposite angles are equal.

Hence Proved.

Q2. Bisectors of interior ∠B and exterior ∠ACD of a Δ ABC intersect at point T. Prove that ∠ BTC = ½ ∠ BAC.

Answer:

Given: △ ABC, produce BC to D, and the bisectors of ∠ABC and ∠ACD meet at point T.



To prove:

∠BTC = ½ ∠BAC

Proof:

In △ABC,∠ACD is an exterior angle.

We know that,

The exterior angle of a triangle is equal to the sum of two opposite angles,

Then,

∠ACD = ∠ABC + ∠CAB

Dividing L.H.S and R.H.S by 2,

⇒ ½ ∠ACD = ½ ∠CAB + ½ ∠ABC

⇒ ∠TCD = ½ ∠CAB + ½ ∠ABC …(1)

[∵CT is a bisector of ∠ACD⇒ ½ ∠ACD = ∠TCD]

We know that,

The exterior angle of a triangle is equal to the sum of two opposite angles,

Then in △ BTC,

∠TCD = ∠BTC +∠CBT

⇒ ∠TCD = ∠BTC + ½ ∠ABC …(2)

[∵BT is the bisector of △ ABC ⇒∠CBT = ½ ∠ABC]

From equations (1) and (2),

We get,

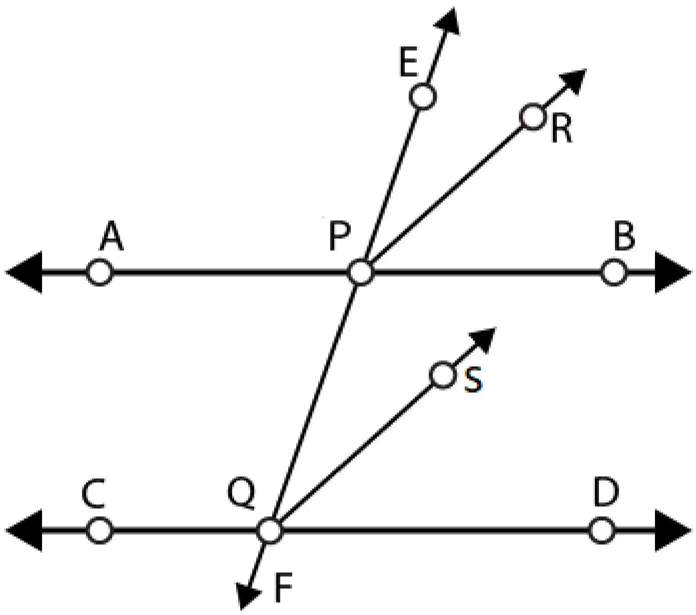
½ ∠CAB + ½ ∠ABC = ∠BTC + ½ ∠ABC

⇒ ½ ∠CAB = ∠BTC or ½ ∠BAC = ∠BTC

Hence, proved.

Q3. A transversal intersects two parallel lines. Prove that the bisectors of any pair of corresponding angles so formed are parallel.

Answer:



Let,

AB ∥ CD

EF be the transversal passing through the two parallel lines at P and Q, respectively.

PR and QS are the bisectors of ∠EPB and ∠PQD.

We know that the corresponding angles of parallel lines are equal,

So, ∠EPB = ∠PQD

½ ∠EPB = ½ ∠PQD

∠EPR = ∠PQS

But, we also know that they are corresponding angles of PR and QS

Since the corresponding angles are equal,

We have,

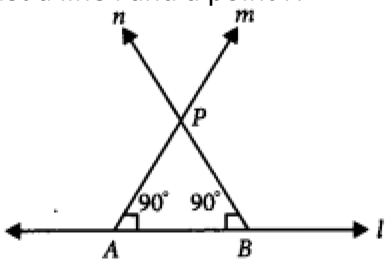
PR ∥ QS

Hence Proved.

Q4. Prove that through a given point, we can draw only one perpendicular to a given line. [Hint: Use proof by contradiction].

Answer:

Let a line l and a point P.



Also let m and n are two lines passing through P and perpendicular to l.

In ΔAPB,

∠A + ∠P + ∠B = 180°

[Angle sum property of a triangle]

⇒ 90° + ∠P + 90° = 180°

⇒ ∠P = 180°- 180°

⇒ ∠P = 0°

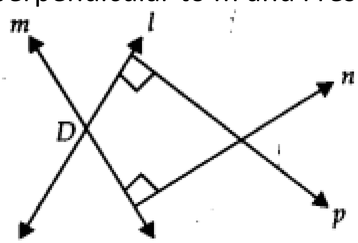
∴ Lines n and m coincide.

Thus, only one perpendicular line can be drawn through a given point on a given line.

Q5. Prove that two lines that are respectively perpendicular to two intersecting lines intersect each other. [Hint: Use proof by contradiction].

Answer:

Let lines l and m be two intersecting lines. Again, let n and p be another two lines which are perpendicular to m and l respectively.



Let us assume that lines n and p are not intersecting, which means they are parallel to each other i.e., n||p …(i)

Since, lines n and p are perpendicular to m and l, respectively.

But from (i), n || p ⇒ l || m, which shows a contradiction.

So, our assumption was wrong.

Thus lines n and p intersect at a point.

Q6.Prove that a triangle must have at least two acute angles.

Answer:

Let ΔABC be a triangle.

We know that the sum of all three angles is 180°.

∴ ∠A + ∠B + ∠C = 180° …(i)

Let us consider the following cases.

Case I: When two angles are 90°.

Suppose two angles ∠B = 90° and ∠C = 90°

So from (i), we get

∠A + 90° + 90° = 180°

⇒ ∠A = 180° -180° = 0

Thus, no triangle is possible.

Case II: When two angles are obtuse.

Suppose ∠B and ∠C are obtuse angles.

From (i), we get ∠A = 180° – (∠B + ∠C)

= 180° – (greater than 180°)

[∵ ∠B + ∠C = more than 90° + more than 90°]

⇒ ∠A = negative angle, which is not possible,

Thus, no triangle is possible.

Case III: When one angle is 90°.

Suppose ∠B = 90°.

From (i), ∠A + ∠B + ∠C= 180°

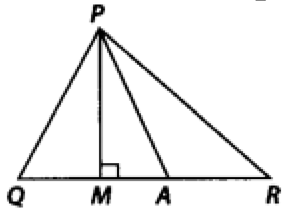
⇒ ∠A + ∠C = 180° – 90° = 90°

So, the sum of the other two angles is 90°, Hence, both angles are acute.

Case IV: When two angles are acute, then the sum of two angles is less than 180°, so that the third angle may be acute or obtuse.

Thus, a triangle must have at least two acute angles.

Q7. In the given figure, ∠Q > ∠R, PA is the bisector of ∠QPR and PM ⊥ QR.
Prove that ∠APM = $\frac{1}{2}$ (∠Q – ∠R).



Answer:

Since, PA is the bisector of ∠QPR.

∴ ∠ QPA = ∠ APR

In ΔPQM, ∠PQM + ∠PMQ + ∠QPM = 180°

[Angle sum property of a triangle]

⇒ ∠PQM + 90° + ∠QPM = 180°

[∵ PM ⊥ QR

⇒ ∠PMQ = 90°]

⇒ ∠PQM = 90° – ∠QPM …(ii)

In ΔPMR,

∠PMR + ∠PRM + ∠RPM = 180°

[Angle sum property of a triangle]

⇒ 90° + ∠PRM + ∠RPM = 180°

[∵ PM ⊥ QR ⇒ ∠PMR = 90°]

⇒ ∠PRM = 180° – 90° – ∠RPM

⇒ ∠PRM = 90° – ∠RPM …(iii)

On subtracting (iii) from (ii), we get

∠Q – ∠R = (90° – ∠QPM) – (90° – ∠RPM)

[∵ ∠PQM = ∠Q and ∠PRM = ∠R]

⇒ ∠Q – ∠R = ∠RPM – ∠QPM

⇒ ∠Q – ∠R = [∠RPA + ∠APM] -[∠QPA – ∠APM]

⇒ ∠Q – ∠R = ∠RPA + ∠APM – ∠QPA + ∠APM

⇒ ∠Q – ∠R = 2∠APM [By using (i)]

∠APM = $\frac{1}{2}$ (∠Q – ∠R)