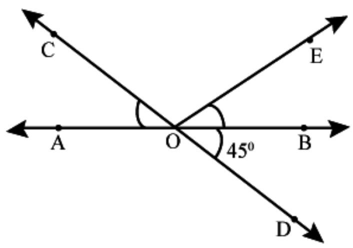
## Exercise 6.1

**Q1. In Fig. 6.13, lines AB and CD intersect at O. If ∠AOC + ∠BOE = 70° and ∠BOD = 40°, find ∠BOE and reflex ∠COE.**

**Solution:**




Let ∠AOC = x and ∠BOE = y.

Then x + y = 70° ( ∠AOC + ∠BOE = 70°)

Let Reflex ∠COE = z

We can see that AB and CD are two intersecting lines, so the pair of angles formed are vertically opposite angles and they are equal.

i.e, ∠AOD = ∠BOC and ∠AOC = ∠BOD.

Since ∠AOC = x and ∠AOC = ∠BOD = 40°

Thus, we can say that x = 40°.

Also we know that,

x + y = 70°

40° + y = 70°

y = 70° - 40° = 30°

∠BOE = 30°

If we consider line AB and ray OD on it, then ∠AOD and ∠BOD are adjacent angles.

∠AOD + ∠BOD = 180°

∠AOD + 40° = 180°

∠AOD = 180° - 40°

= 140°

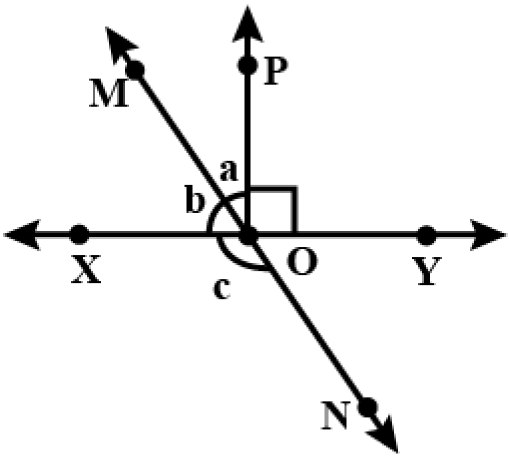
Reflex ∠COE = ∠AOC + ∠AOD + ∠BOD + ∠BOE

= 40° + 140° + 40° + 30°

= 250°

Thus, ∠BOE = 30° and the reflex ∠COE = 250°.

**2. In Fig. 6.14, lines XY and MN intersect at O. If ∠POY = 90° and a : b = 2 : 3, find c.**




**Solution:**

Given: ∠POY= 90° and a : b = 2 : 3.

If two lines intersect with each other, then the vertically opposite angles formed are equal.

Line OP is perpendicular to line XY. Hence ∠POY = ∠POX = 90°

∠POX = ∠POM + ∠MOX

90° = a + b ….(1)

Since a and b are in the ratio 2 : 3 that is,

a = 2x and b = 3x ….(2)

Substituting (2) in (1),

a + b = 90°

2x + 3x = 90°

5x = 90°

x = 90°/5 = 18°

a = 2x = 2 × 18°

a = 36°

b = 3x = 3 × 18°

b = 54°

Also , ∠MOY= ∠MOP + ∠POY

= a + 90°

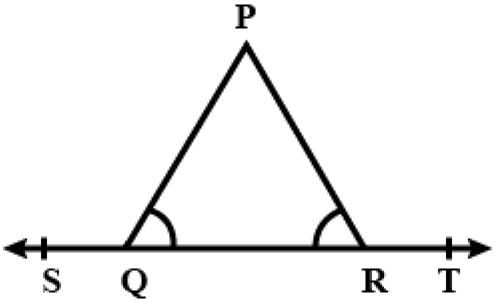
= 36° + 90° = 126°

Lines MN and XY intersect at point O and the vertically opposite angles formed are equal.

∠XON = ∠MOY

c = 126°

**3. In Fig. 6.15, ∠PQR = ∠PRQ, then prove that ∠PQS = ∠PRT.**




**Solution:**

Given: ∠PQR = ∠PRQ

To prove: ∠PQS = ∠PRT

We know that, if a ray stands on a line, then the sum of adjacent angles formed is 180°.

Let ∠PQR = ∠PRQ = a.

Let ∠PQS = b and ∠PRT = c.

Lines ST and PQ intersect at point Q, hence the sum of adjacent angles ∠PQS and ∠PQR is 180°.

∠PQS + ∠PQR = 180°

b + a = 180°

b = 180° - a….(1)

Lines ST and PR intersect at point R, hence the sum of adjacent angles ∠PRQ and ∠PRT is 180°.

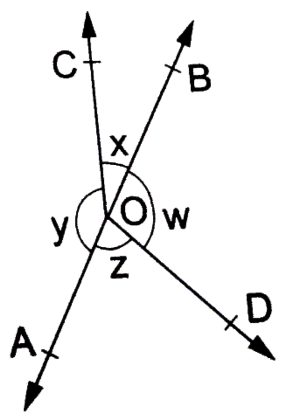
∠PRQ + ∠PRT = 180°

a + c = 180°

c = 180° - a ….(2)

From equations (1) and (2), it is clear that b = c. Hence ∠PQS = ∠PRT is proved.

**4. In Fig. 6.16, if x + y = w + z, then prove that AOB is a line.**




**Solution:**

Given: x + y = w + z

To prove: AOB is a line.

We know that if the sum of two adjacent angles is 180°, then the non-common arms of the angles form a line.

From the figure we can see that,

(x + y) + (w + z) = 360° (complete angle)

It is given that (x + y) = (w + z),

Hence (x + y) + (w + z) = 360° can be written as (x + y) + (x + y) = 360°

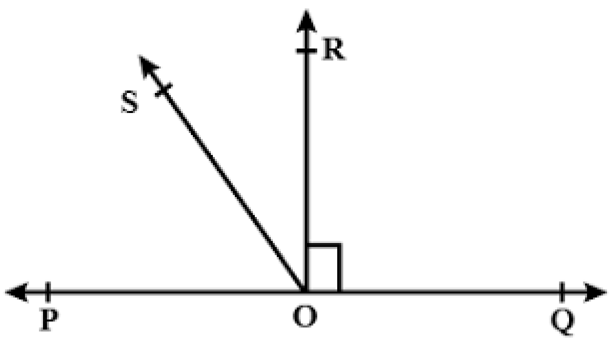
2x + 2y = 360°

2(x + y) = 360°

x + y = 360°/2 = 180°

Since the sum of adjacent angles, x and y with OA and OB as the non-common arms is 180° we can say that AOB is a line.

**5. In Fig. 6.17, POQ is a line. Ray OR is perpendicular to line PQ. OS is another ray lying between rays OP and OR. Prove that ∠ROS = ½ (∠QOS – ∠POS).**




**Solution:**

Given: OR is perpendicular to PQ. ∠ROQ = ∠ROP = 90°.

To prove: ∠ROS = 1/2(∠QOS - ∠POS).

When a ray intersects a line, the sum of adjacent angles so formed is 180°.

Let ∠ROS = a, ∠POS = b and ∠SOQ = c.

To prove that: a = 1/2(c - b).

Since ∠ROQ = ∠ROP = 90°,

We can say, ∠POS + ∠SOR = ∠POR

b + a = 90°.....(1)

Line PQ is intersected by ray OS.

Hence ∠POS + ∠SOQ = b + c = 180°

b + c = 180°..... (2)

From equation (1), we get a + b = 90°

Multiplying by 2 on both sides we get,

2(a + b) = 2 × 90°

2(a + b) = 180°.....(3)

Comparing equations (3) and (2),

2(a + b) = b + c

2a + 2b = b + c

2a = b + c - 2b

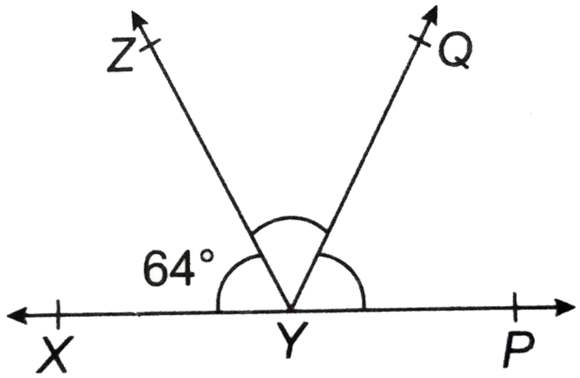
2a = c - b

a = 1/2 (c - b)

∴ ∠ROS = 1/2 (∠QOS - ∠POS)

**6. It is given that ∠XYZ = 64° and XY is produced to point P. Draw a figure from the given information. If ray YQ bisects ∠ZYP, find ∠XYQ and reflex ∠QYP.**

**Solution:**




Given: ∠XYZ = 64° and Ray YQ bisects ∠PYZ.

To Find: ∠XYQ and Reflex ∠QYP

When a ray intersects a line, the sum of adjacent angles formed is 180°.

With the given information in the question, we can come up with this diagram.

Ray YQ bisects ∠ZYP.

Let, ∠ZYQ = ∠QYP = a.

We can see from the figure that PX is a line and YZ is a ray intersecting at point Y and the sum of adjacent angles so formed is 180°.

Hence ∠ZYP + ∠ZYX = 180°

∠ZYQ + ∠QYP + ∠ZYX = 180° [Since, ∠ZYP = ∠ZYQ + ∠QYP]

a + a + 64° = 180°

2a + 64° = 180°

2a = 180° - 64° = 116°

a = 116°/2 = 58°

∴ Then ∠XYQ = ∠XYZ + ∠ZYQ

∠XYQ = a + 64°

⇒ ∠XYQ = 58° + 64° = 122°.

∠XYQ = 122°

As ∠QYP = a,

Thus, Reflex ∠QYP = 360° - a

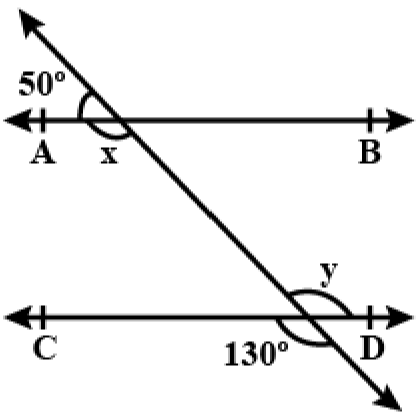
⇒ 360° - 58° = 302°

Reflex ∠QYP = 302°.

Thus, we have ∠XYQ = 122° and Reflex ∠QYP = 302°.

## Exercise 6.2

**Q1. In Fig. 6.28, find the values of x and y and then show that AB || CD.**



**Solution:**

When two lines intersect, vertically opposite angles are formed at the point of intersection which is equal.

Also, when a ray intersects a line, the sum of adjacent angles formed is 180° as it forms a linear pair.

If a transversal intersects two lines such that a pair of alternate interior angles are equal, then the two lines are parallel to each other.

Line CD is intersecting with line P. Hence the vertically opposite angles so formed are equal.

Thus, y = 130°.

Similarly, line AB intersects with line P forming a linear pair. Hence the sum of adjacent angles formed is 180°.

x + 50° = 180°

x = 180° - 50°

x = 130°

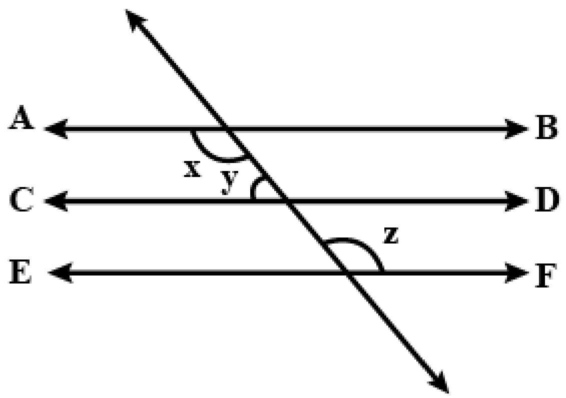
We know that, if a transversal intersects two lines such that a pair of alternate interior angles are equal, then the two lines are parallel.

Here we can see that the pair of alternate angles formed when lines AB and CD are intersected by transversal P are equal.

i.e, x = y = 130°. So we can say the two lines AB and CD are parallel.

Hence AB || CD is proved.

**2. In Fig. 6.29, if AB || CD, CD || EF and y : z = 3 : 7, find x.**



**Solution:**

Given: AB || CD, CD || EF and y : z = 3 : 7

To find: The value of x

When two parallel lines are cut by a transversal, co-interior angles formed are supplementary.

Also, we know that lines which are parallel to the same line are parallel to each other.

Thus, If AB || CD, CD || EF, we can say AB || EF.

Therefore, the angles x and z are alternate interior angles and hence are equal.

x = z.......(1)

AB and CD are parallel lines cut by a transversal. So the co-interior angles formed are supplementary.

$x + y = 180°$.

Since x = z,

We get $y + z = 180°$......... (2)

Let, y = 3a, z = 7a [Since, y : z = 3 : 7]

Substituting the values in equation (2),

$3a + 7a = 180°$

$10a = 180°$

$a = 180°/10$

$a = 18°$

$\therefore y = 3a = 3 \times 18 = 54°$

$y = 54°$

$\therefore x + y = 180°$

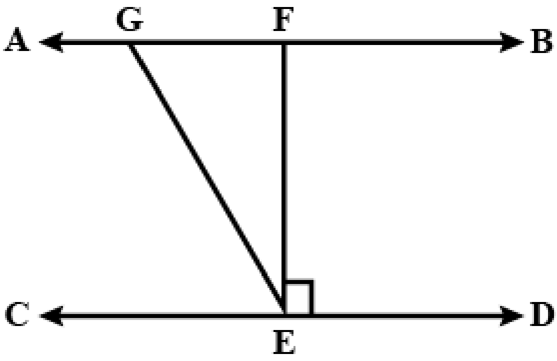
$x + 54° = 180°$

$x = 180° - 54°$

$x = 126°$

Thus, $x = 126°$

**Q3. In Fig. 6.30, if AB || CD, EF ⊥ CD and ∠GED = 126°, find ∠AGE, ∠GEF and ∠FGE.**




**Solution:**

Given: AB || CD, EF ⊥ CD and ∠GED =126°

To Find: ∠AGE, ∠GEF and ∠FGE

When two lines intersect, the sum of adjacent angles is supplementary.

When two parallel lines are cut by a transversal, alternate interior angles formed are equal.

Let ∠AGE = x, ∠GEF = y and ∠FGE = z.

From the figure, we can see that,

∠GED = ∠GEF + ∠FED

∠GEF = ∠GED - ∠FED

y = 126° - 90° [ Since, ∠GED = 126° and ∠FED = 90°]

y = 36°

Thus, ∠GEF = y = 36°

AB and CD are parallel lines cut by a transversal, thus the pair of alternate interior angles formed are equal.

∠AGE = ∠GED

Thus, ∠AGE = x = 126°

Line AB is intersected by line GE where x and z forms a linear pair.

x + z = 180°

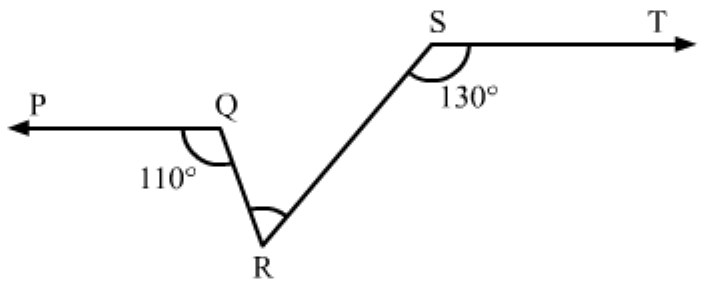
126° + z = 180°

z = 180° - 126° = 54°

Thus, ∠FGE = z = 54°

**Q4. In Fig. 6.31, if PQ || ST, ∠PQR = 110° and ∠RST = 130°, find ∠QRS.**

**[Hint: Draw a line parallel to ST through point R.]**




**Answer:**

Given: PQ || ST, ∠PQR = 110° and ∠RST = 130°

To Find: ∠QRS

Lines which are parallel to the same line are parallel to each other.

When two parallel lines are cut by a transversal, co-interior angles formed are supplementary.

Construction: Draw a line AB parallel to ST through point R. Since AB || ST and we know that PQ || ST. Thus, AB || PQ.

Let ∠SRQ = x, ∠SRB = y and ∠QRA = z

Lines ST and AB are parallel with transversal SR intersecting them. Therefore, the co-interior angles formed are supplementary.

∠RST + ∠SRB = 180°

130° + y = 180°

y = 180° - 130° = 50°

Thus, ∠SRB = y = 50°

Similarly, lines PQ and AB are parallel with transversal QR intersecting the two lines. Therefore, the co-interior angles are supplementary.

∠PQR + ∠QRA = 180°

110° + z = 180°

z = 180° - 110° = 70°

Thus, ∠QRA = z = 70°

AB is a line, RQ and RS are rays on AB. Hence,

∠QRA + ∠QRS + ∠SRB = 180°

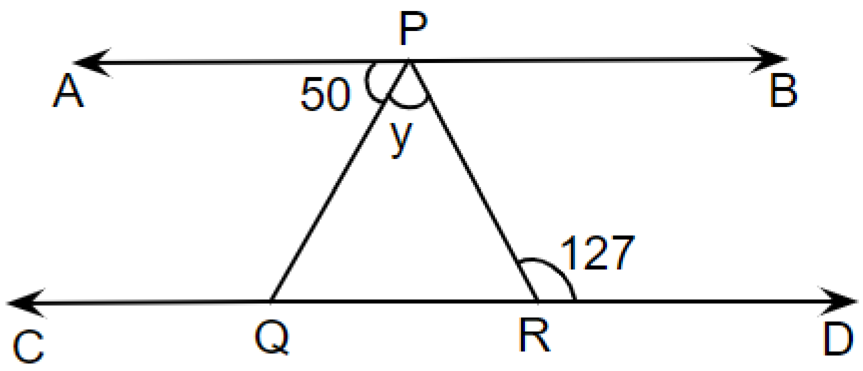
70° + x + 50° = 180°

120° + x = 180°

x = 180° - 120° = 60°

Thus, ∠QRS = x = 60°.

**Q5. In Fig. 6.32, if AB || CD, ∠APQ = 50° and ∠PRD = 127°, find x and y.**




**Answer:**

Given: AB || CD, ∠APQ = 50° and ∠PRD = 127°

To Find: x and y

When a ray intersects a line, the sum of adjacent angles formed is 180°.

When two parallel lines are cut by a transversal, alternate interior angles formed are equal.

AB and CD are parallel lines cut by transversal PQ hence the alternate interior angles formed are equal.

∠APQ = ∠PQR and hence x = 50°.

Similarly, AB and CD are parallel lines cut by transversal PR hence the alternate angles formed are equal.

∠APR = ∠PRD = 127°

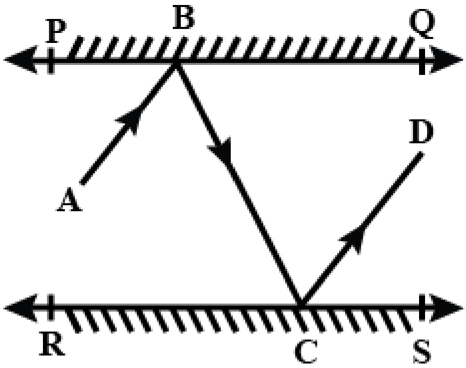
∠APQ + ∠QPR = ∠PRD = 127°

50° + y = 127°

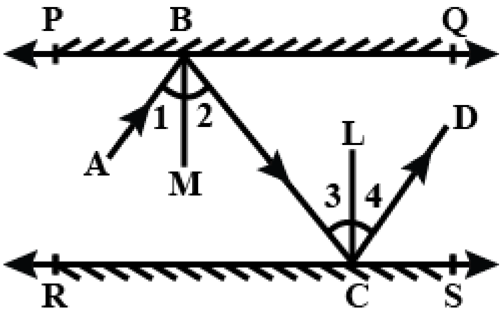
y = 127° - 50°

y = 77°

**Q6. In Fig. 6.33, PQ and RS are two mirrors placed parallel to each other. An incident ray AB strikes the mirror PQ at B, the reflected ray moves along the path BC and strikes the mirror RS at C and again reflects back along CD. Prove that AB || CD.**




**Answer:**




Given: PQ || RS

To prove: AB || CD

When two parallel lines are cut by a transversal, alternate angle formed are equal.

In optics the angle of incidence (the angle which an incident ray makes perpendicular to the surface at the point of incidence) and the angle of reflection (the angle formed by the reflected ray perpendicular to the surface at the point of incidence) are equal.

Draw perpendicular lines BL and CM at the point of incidence on both mirrors. Since PQ and RS are parallel to each other, perpendiculars drawn are also parallel i.e, BL ∥ CM.

Since BC is a transversal to lines BL and CM, alternate interior angles are equal.

Hence, ∠LBC = ∠BCM = x (say).... (1)

Bylaws of reflection, at the first point of incidence B on mirror PQ, we get,

∠ABL = ∠LBC = x

∴ ∠ABC = ∠ABL + ∠LBC

= x + x

∴ ∠ABC = 2x....(2)

Bylaws of reflection, at the second point of incidence C on mirror RS, we get,

∠MCD = ∠BCM = x

∴ ∠BCD = ∠BCM + ∠MCD

= x + x

∠BCD = 2x.....(3)

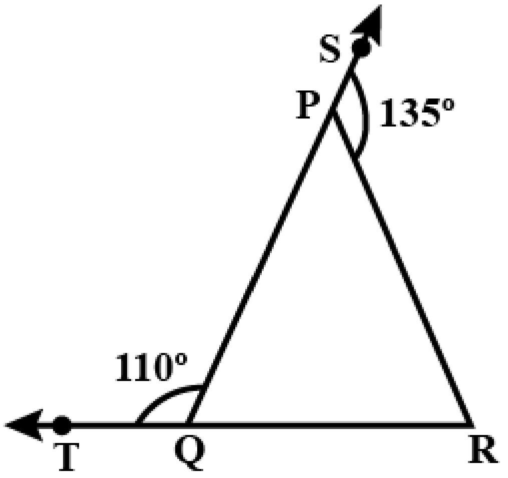
From equations (2) and (3), we get ∠ABC = ∠BCD which are alternate interior angles for the lines AB and CD and BC as the transversal.

We know that, if a transversal intersects two lines such that a pair of alternate interior angles are equal, then the two lines are parallel.

Since. alternate interior angles are equal, we can say AB || CD.

## Exercise 6.3

**Q1. In Fig. 6.39, sides QP and RQ of ΔPQR are produced to points S and Respectively. If ∠SPR = 135° and ∠PQT = 110°, find ∠PRQ.**




**Answer:**

Given: ∠SPR = 135° and ∠PQT = 110°

To find: ∠PRQ

We know that if non-common arms of two adjacent angles form a line, then these angles are called linear pair angle and their sum is equal to 180°.

If the sum of two adjacent angles is 180° then the two non-common arms of the angles form a line.

According to the Angle sum property of a triangle, the sum of the interior angles of a triangle is 180°.

∠SPR + ∠QPR = 180° [Linear pair]

135° + ∠QPR = 180°

∠QPR = 180° - 135°

∠QPR = 45°.....(i)

∠PQT + ∠PQR = 180° [Linear pair]

110° + ∠PQR = 180°

∠PQR = 180° - 110°

∠PQR = 70°.....(ii)

Now,

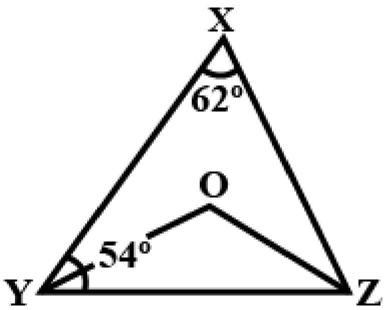
∠PQR + ∠QPR + ∠PRQ = 180° [Angle sum property of a triangle]

70°+ 45° + ∠PRQ = 180° [from (i) and (ii)]

∠PRQ = 180° - 115°

∠PRQ = 65°

**Q2. In Fig. 6.40, ∠X = 62°, ∠XYZ = 54°. If YO and ZO are the bisectors of ∠XYZ and ∠XZY, respectively of Δ XYZ, find ∠OZY and ∠YOZ.**




**Answer:**

Given: ∠X = 62°, ∠XYZ = 54°, and YO and ZO are the bisectors of ∠XYZ and ∠XZY respectively.

To find: ∠OZY and ∠YOZ

According to the angle sum property of a triangle, sum of the interior angles of a triangle is 180°.

Consider △XYZ

∠X + ∠XYZ + ∠Z = 180° [Angle sum property of a triangle]

62° + 54° + ∠Z = 180°

∠Z = 180° - 116°

∠Z = 64°

Now, OZ is the angle bisector of ∠XZY

Thus, ∠OZY = (1/2) of ∠XZY = 1/2 × 64° = 32° ..........(i)

Similarly, OY is the angle bisector of ∠XYZ

Thus, ∠OYZ = (1/2) of ∠XYZ = 1/2 × 54° = 27°......... (ii)

Now, in △OYZ

∠OYZ + ∠OZY + ∠YOZ = 180° [Angle sum property of a triangle]

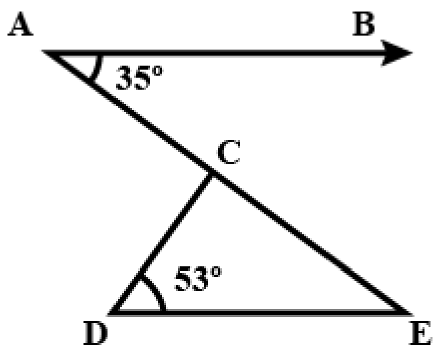
27° + 32° + ∠YOZ = 180° [from (i) and (ii)]

∠YOZ = 180° - 59°

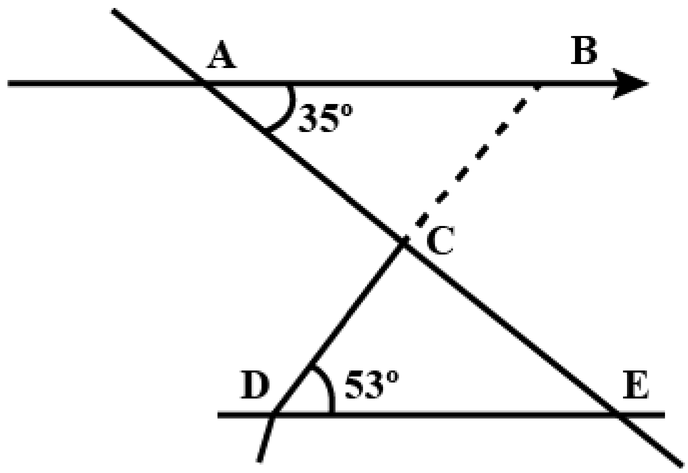
∠YOZ = 121°

Hence, ∠OZY = 32° and ∠YOZ = 121°

**Q3. In Fig. 6.41, if AB || DE, ∠BAC = 35° and ∠CDE = 53°, find ∠DCE.**




**Answer:**




Given: AB || DE, ∠BAC = 35° and ∠CDE = 53°

To find: ∠DCE

We know that when two parallel lines are cut by a transversal, alternate interior angles formed are equal.

According to angle sum property of a triangle, sum of the interior angles of a triangle is 180°.

Since, AB || DE and AE is the transversal,

∠DEC = ∠BAC [Alternate interior angles]

Thus, ∠DEC = 35°

Now, in △CDE

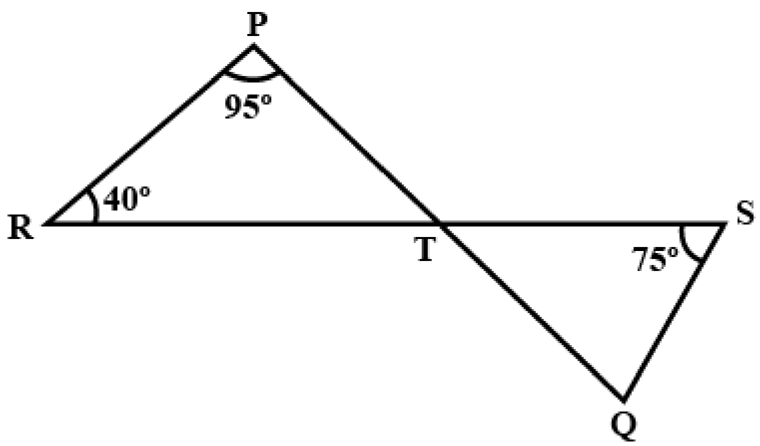
∠CDE + ∠DEC + ∠DCE = 180° [Angle sum property of a triangle]

53° + 35° + ∠DCE = 180°

∠DCE = 180° - 88°

Thus, we have ∠DCE = 92°.

**Q4. In Fig. 6.42, if lines PQ and RS intersect at point T, such that ∠PRT = 40°, ∠RPT = 95° and ∠TSQ = 75°, find ∠SQT.**




**Answer:**

Given: ∠PRT = 40° , ∠RPT = 95° and ∠TSQ = 75°

To find: ∠SQT

We know that when two lines intersect each other at a point then there are two pairs of vertically opposite angles formed that are equal.

According to the angle sum property of a triangle, sum of the interior angles of a triangle is 360°.

In △PRT,

∠PTR + ∠PRT + ∠RPT = 180° [Angle sum property of a triangle]

∠PTR + 40° + 95° = 180°

∠PTR = 180° - 135°

∠PTR = 45°

Now,

∠QTS = ∠PTR [Vertically opposite angles]

∠QTS = 45°...... (i)

In △TSQ,

∠QTS + ∠TSQ + ∠SQT = 180° [Angle sum property of a triangle]

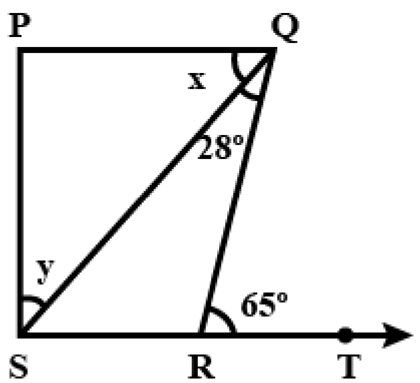
45° + 75° + ∠SQT = 180° [From (i)]

∠SQT = 180° - 120°

∠SQT = 60°

Hence, ∠SQT = 60°

**5. In Fig. 6.43, if PQ ⊥ PS, PQ || SR, ∠SQR = 28° and ∠QRT = 65°, then find the values of x and y.**




**Answer:**

Given: PQ ⊥ PS, PQ || SR, ∠SQR = 28° and ∠QRT = 65°

To find: x and y

We know when two parallel lines are cut by a transversal, alternate interior angles formed are equal.

According to the angle sum property of a triangle, sum of the interior angles of a triangle is 360°.

Since, PQ || SR and QR is the transversal,

∠PQR = ∠QRT [Alternate interior angles]

∠PQS + ∠SQR = ∠QRT [From figure]

$x + 28° = 65°$

$x = 65° - 28°$

$x = 37°$

Now, in △PQS

∠PQS + ∠PSQ + ∠QPS = 180° [Angle sum property of a triangle]

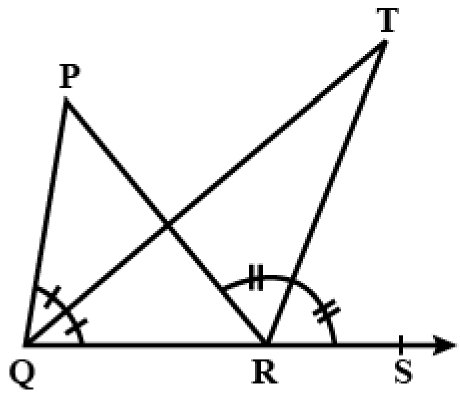
37° + y + 90° = 180° [Since. ∠QPS = 90°]

$y = 180° - 127°$

$y = 53°$

Hence, x = 37° and y = 53°

**6. In Fig. 6.44, the side QR of ΔPQR is produced to a point S. If the bisectors of ∠PQR and ∠PRS meet at point T, then prove that ∠QTR = ½ ∠QPR.**




**Answer:**

Given: TQ and TR are the bisectors of ∠PQR and ∠PRS respectively

To Prove: ∠QTR = 1/2 ∠QPR

According to the exterior angle theorem of a triangle, if a side of a triangle is produced, then the exterior angle so formed is equal to the sum of the two interior opposite angles.

∠PRS = 2∠TRS.......(i) [Since TR is the angle bisector of ∠PRS]

∠PQR = 2∠TQR .........(ii) [Since TQ is the angle bisector of ∠PQR]

Now, in △TQR

∠TRS = ∠TQR +∠QTR [Exterior angle theorem of a triangle]

∠QTR = ∠TRS - ∠TQR.......... (iii)

Similarly, in △PQR

∠PRS = ∠QPR + ∠PQR [Exterior angle theorem of a triangle]

2∠TRS = ∠QPR + 2∠TQR [From (i) and (ii)]

∠QPR = 2∠TRS - 2∠TQR

∠QPR = 2(∠TRS - ∠TQR)

∠QPR = 2∠QTR [From (iii)]

∠QTR = 1/2 ∠QPR

Hence proved, ∠QTR = 1/2 ∠QPR.