Exercise 10.1:

Q1. Fill in the blanks:

(i) The centre of a circle lies in____of the circle. (exterior / interior)

(ii) A point, whose distance from the centre of a circle is greater than its radius lies in____of the circle. (exterior / interior)

(iii) The longest chord of a circle is a_____of the circle.

iv) An arc is a______when its ends are the ends of a diameter.

(v) Segment of a circle is the region between an arc and______of the circle.

(vi) A circle divides the plane, on which it lies, in_______parts.

Answer:

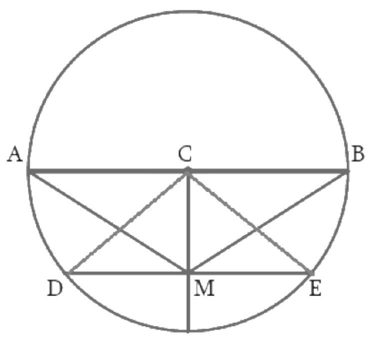
(i) The center of the circle lies in interior of the circle. (exterior / interior)

Reasoning: The collection of all points in a plane, which is at a fixed distance from a fixed point in the plane, is called a circle. The fixed point is the center of the circle.

(ii) A point, whose distance from the center of the circle is greater than its radius lies in exterior of the circle. (exterior / interior)

Reasoning: The collection of all points in a plane, which is at a fixed distance from a fixed point in the plane is called a circle. The fixed point is the center of the circle. Fixed distance is the radius of the circle. Any point outside the circle will have a greater distance compared to the radius.

(iii) The longest chord of the circle is a diameter of the circle.



Reasoning: Let us check by drawing a random chord DE and diameter AB in the circle.

AC = CD = CE = BC = radius AB = 2 × radius.

In ΔDCE, DE < DC + CE (sum of two sides of a triangle should be greater than the third side) DE < 2 × radius

DE < diameter

Thus, we know that any chord that is drawn randomly (without passing through the center) will be shorter than the diameter. Thus, the diameter is the longest chord in the circle.

(iv) An arc is a semicircle when its ends are the ends of a diameter.

Reasoning: We know that diameter is the longest chord in the circle. Diameter divides the circle into 2 equal halves or arcs. When two arcs are equal, each is a semicircle.

(v) Segment of a circle is the region between an arc and chord of the circle.

Reasoning: The region between a chord and either of its arcs is called a segment of the circular region or simply a segment of the circle.

(vi) A circle divides the plane, on which it lies, in three parts.

Reasoning: A circle divides the plane on which it lies into three parts. They are: (i) inside the circle, which is also called the interior of the circle; (ii) the circle and (iii) outside the circle, which is also called the exterior of the circle.

Q2. Write True or False: Give reasons for your answers.

(i) Line segment joining the centre to any point on the circle is a radius of the circle.

(ii) A circle has only finite number of equal chords.

(iii) If a circle is divided into three equal arcs, each is a major arc.

(iv) A chord of a circle, which is twice as long as its radius, is a diameter of the circle.

(v) Sector is the region between the chord and its corresponding arc.

(vi) A circle is a plane figure.

Answer:

(i) Line segment joining the center to any point on the circle is the radius of the circle.

Answer: True

Reasoning: The collection of all points in a plane, which are at a fixed distance from a fixed point in the plane is called a circle. The fixed point is the center of the circle. Fixed distance is the radius of the circle.

(ii) A circle has only a finite number of equal chords.

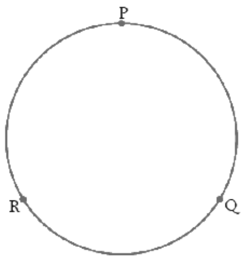
Answer: False

Reasoning: There are infinite points on the circle and hence infinite chords can be drawn between these points.

(iii) If a circle is divided into three equal arcs, each is a major arc.

Answer: False

Reasoning: If PQ is a minor arc, then QRP is a major arc and it should be greater than a semicircular arc. If there are three arcs none of it can be a major arc.



(iv) A chord of a circle, which is twice as long as its radius, is the diameter of the circle.

Solution: True

Reasoning: Draw a chord that passes through the center of the circle. We can see that this chord is twice the length of the radius of the circle. This is called the diameter of the circle.

(v) Sector is the region between the chord and its corresponding arc.

Solution: False

Reasoning: The region between an arc and the two radii, joining the center to the endpoints of the arc is called a sector.

(vi) A circle is a plane figure.

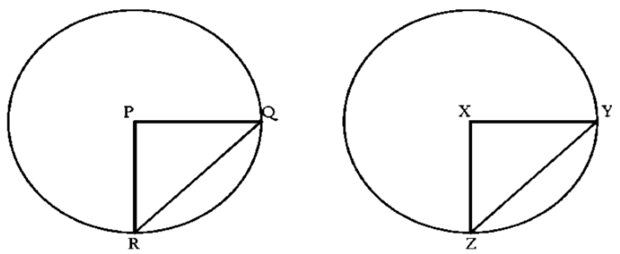
Solution: True

Reasoning: A circle is a 2-dimensional figure. So, circle is a plane figure.

Exercise 10.2:

1. Recall that two circles are congruent if they have the same radii. Prove that equal chords of congruent circles subtend equal angles at their centres.

Answer:



Let QR and YZ be the equal chords of 2 congruent circles.

Then, QR = YZ

We need to prove that they subtend equal angles at the center. That is, ∠QPR = ∠YXZ

We know that the radii of both circles are equal. So, we get: PR = PQ = XZ = XY

Consider the 2 triangles, ΔPQR and ΔXYZ.

PQ = XY (Radii are equal)

PR = XZ (Radii are equal)

QR = YZ (Chords are equal)

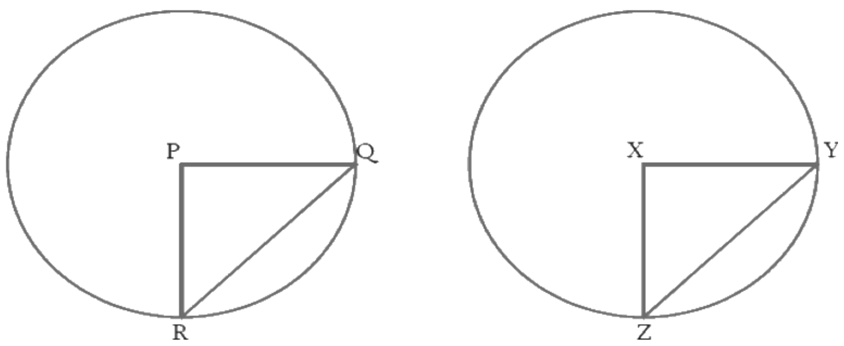
By SSS congruency, ΔPQR is congruent to ΔXYZ.

So, by CPCT (Corresponding parts of congruent triangles), we get ∠QPR =∠YXZ.

Hence, proved that equal chords of congruent circles subtend equal angles at their centers.

2. Prove that if chords of congruent circles subtend equal angles at their centres, then the chords are equal.

Answer: Using equal angles at the centers and the fact that circles are congruent, we prove the statement using Side-Angle-Side (SAS criteria) and corresponding parts of congruent triangles (CPCT).



Draw chords QR and YZ in two congruent circles as shown above. Join the radii PR, PQ, and XY, XZ respectively.

Given that chords subtend equal angles at the center. So, ∠QPR = ∠YXZ.

We need to prove that chords are equal, that is, QR = YZ

Since the circles are congruent, their radii will be equal.

PR = PQ = XZ = XY

Consider the two triangles ΔPQR and ΔXYZ.

PQ = XY (Radii are equal)

∠QPR = ∠YXZ (Chords subtend equal angles at center)

PR = XZ (Radii are equal)

By SAS criteria, ΔPQR is congruent to ΔXYZ.

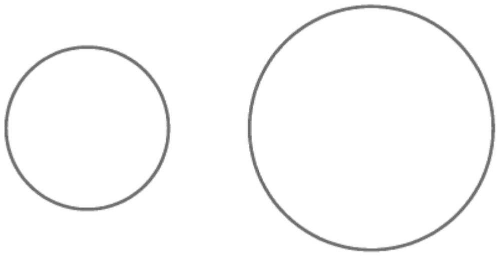
So, QR = YZ (Corresponding parts of congruent triangles)

Hence proved if chords of congruent circles subtend equal angles at their center then the chords are equal.

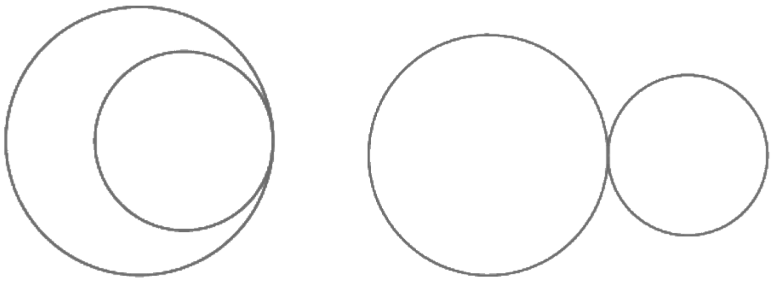
Exercise 10.3:

1. Draw different pairs of circles. How many points does each pair have in common? What is the maximum number of common points?

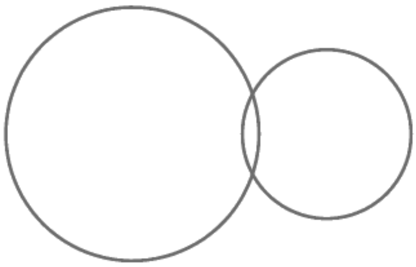
Answer: In this, there are no common points.



In this, there is only one common point.



In this, there are two common points.

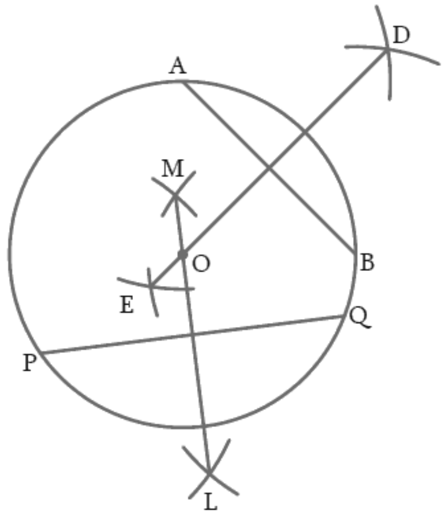


In a situation where two congruent circles are superimposed on each other, it can be understood as if we are drawing the same circle two times.

It is therefore concluded, there can be a maximum of two common points for different pairs of circles.

2. Suppose you are given a circle. Give a construction to find its centre.

Answer:



Construction:

Step 1: Draw a circle with a convenient radius.

Step 2: Draw 2 chords AB and PQ of any length.

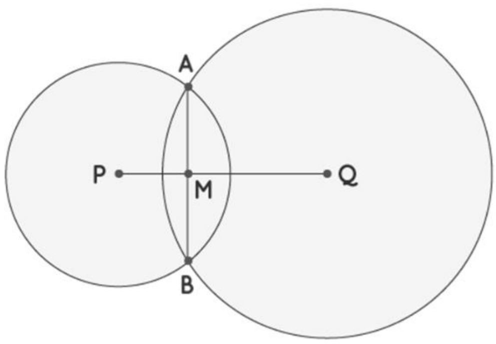
Step 3: With A as center and radii more than half the length of AB, draw two arcs on opposite sides of chord AB. With the same radius and with B as center, draw two arcs cutting the former arcs. Join the line. Now DE is the perpendicular bisector of AB.

Step 4: With P as center and radii more than half the length of PQ, draw two arcs on opposite sides of chord PQ. With the same radius and with Q as the center, draw two arcs cutting the former arcs. Join the line. Now LM is the perpendicular bisector of PQ.

Step 5: As the center of the circle should lie both on DE and LM, it is obvious that the intersection points of DE and LM is the center of circle. Mark the intersection points as O.

3. If two circles intersect at two points, prove that their centres lie on the perpendicular bisector of the common chord.

Answer:



Let the two circles with centre P and Q intersect at points A and B.

Join AB. AB is the common chord.

Join PQ. AB and PQ bisect each other at M.

Let M be the midpoint of AB.

Hence, PM ⊥ AB [Since, the line drawn through the centre of a circle to bisect a chord is perpendicular to the chord]

⇒ ∠PMA = 90º

Now, since M is the midpoint of AB

Hence, QM ⊥ AB

⇒ ∠QMA = 90º

Thus, ∠PMQ = ∠PMA + ∠QMA = 90º + 90º = 180º

Hence PMQ is a straight line and PMQ ⊥ AB

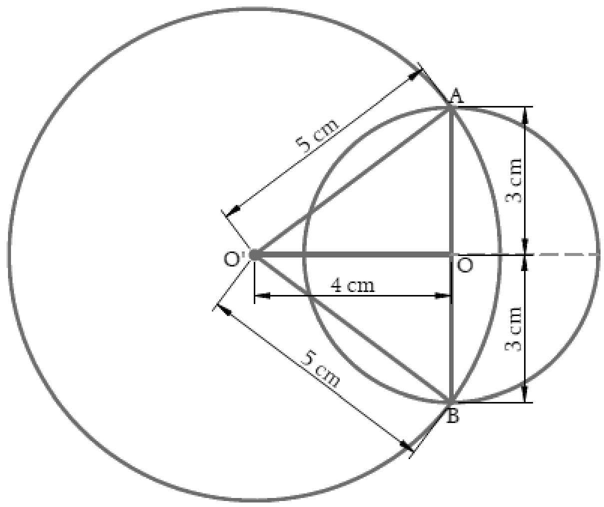
Therefore, PMQ is the perpendicular bisector of the common chord AB and passes through the two centers P and Q.

So, the centres lie on the perpendicular bisector of the common chords.

Exercise 10.4:

1. Two circles of radii 5 cm and 3 cm intersect at two points, and the distance between their centres is 4 cm. Find the length of the common chord.

Answer: The perpendicular bisector of the common chord passes through the centers of both circles.



Given that the circles intersect at two points, so we can draw the above figure. Let AB be the common chord. Let O and O’ be the centers of the circles, respectively.

O’A = 5 cm, OA = 3 cm

OO’ = 4 cm [Given distance between the centres is 4cm]

Since the radius of the bigger circle is more than the distance between the 2 centers, we can say that the center of the smaller circle lies inside, the bigger circle itself.

OO’ is the perpendicular bisector of AB.

So, OA = OB = 3 cm

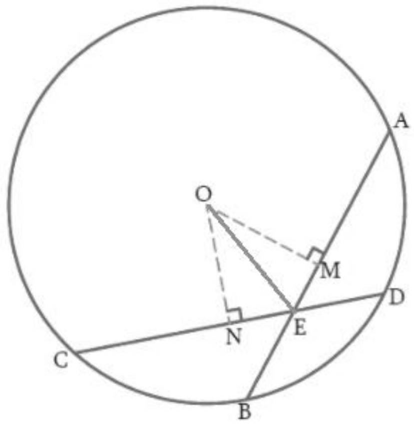
AB = 3 cm + 3 cm = 6 cm [Since, O is the mid point of AB]

The length of the common chord is 6 cm.

It is also evident that the common chord is the diameter of the smaller circle.

2. **If two equal chords of a circle intersect within the circle, prove that the segments of one chord are equal to corresponding segments of the other chord.**

Answer:



Let AB and CD be the 2 equal chords. AB = CD. Let the chords intersect at point E. Join OE.

To prove AE = CE and BE = DE.

Draw perpendiculars from the center O to the chords. This Perpendicular bisects the chord AB at M and CD at N.

Thus, AM = MB = CN = DN ……(1)

In ΔOME and ΔONE

∠M = ∠N = 90°

OE = OE

OM = ON (Equal chords are equidistant from the center.)

By RHS criteria, ΔOME and ΔONE are congruent.

So by CPCT, ME = NE ….. (2)

We know that: CE = CN + NE and AE = AM + ME

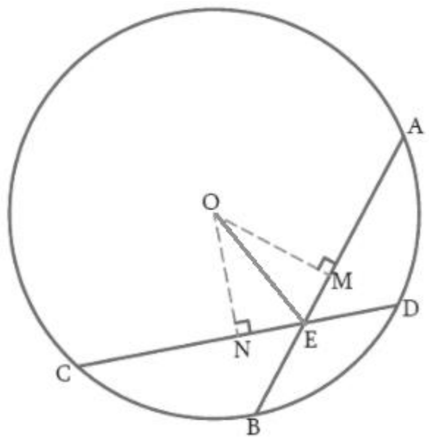
From (1) and (2), it is evident CE = AE

DE = CD - CE and BE = AB - AE

AB and CD are equal, CE and AE are equal. So, DE and BE are also equal. It is proved corresponding segments of equal chords are equal.

3. If two equal chords of a circle intersect within the circle, prove that the line joining the point of intersection to the centre makes equal angles with the chords

Answer:



Let AB and CD be the two equal chords. AB = CD.

Let the chords intersect at point E. Join OE.

Draw perpendiculars from the center O to the chords.

The Perpendicular bisects the chord AB at M and CD at N.

To prove: ∠OEN = ∠OEM.

In ΔOME and ΔONE,

∠M = ∠N = 90°

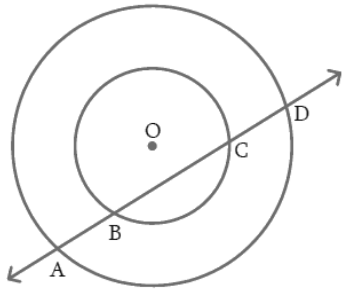
OE = OE

OM = ON (Equal chords are equidistant from the center.)

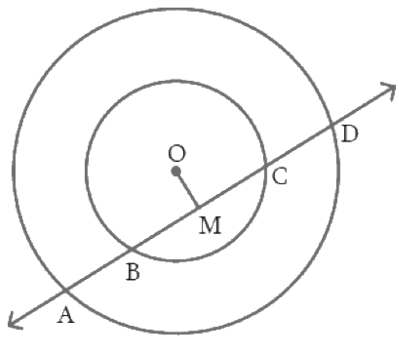
By RHS criteria, ΔOME and ΔONE are congruent. So, by CPCT, ∠OEN = ∠OEM

Hence proved that the line joining the point of intersection of two equal chords to the center makes equal angles with the chords.

4. If a line intersects two concentric circles (circles with the same centre) with centre O at A, B, C and D, prove that AB = CD (see Fig. 10.25).



Answer:



Draw a perpendicular from the center of the circle OM to the line AD.

We can see that BC is the chord of the smaller circle, and AD is the chord of the bigger circle.

We know that perpendicular drawn from the center of the circle bisects the chord.

∴ BM = MC ... (1)

and, AM = MD ... (2)

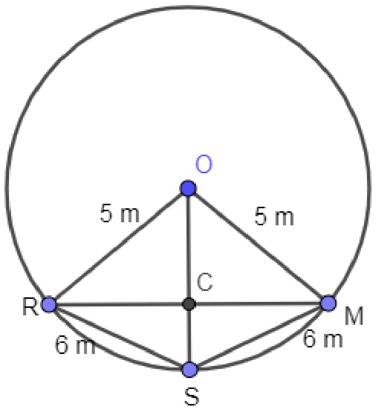
Subtracting (2) from (1), we obtain

AM − BM = DM − CM

∴ AB = CD

5. Three girls Reshma, Salma and Mandip are playing a game by standing on a circle of radius 5m drawn in a park. Reshma throws a ball to Salma, Salma to Mandip, Mandip to Reshma. If the distance between Reshma and Salma and between Salma and Mandip is 6m each, what is the distance between Reshma and Mandip?

Answer: Perpendicular from center to either of the chord bisects the chord. Using this fact and the Pythagoras theorem, we can find the distance between Reshma and Mandip.



Let O be the center of the circle, and R, M and S denote Reshma, Mandip, and Salma respectively.

Draw a perpendicular OA to RS from O. Then RA = AS = 3 m.

Using the Pythagoras theorem, we get OA = 4 m.

We can see that quadrilateral ORSM takes the shape of a kite. (Because OR = OM and RS = SM).

We know that the diagonals of a kite are perpendicular, and the main diagonal bisects the other diagonal.

$\angle$RNS will be 90° and RN = NM

Area of ΔORS = 1/2 × RS × OA

= 1/2 × 6 × 4

= 12 …(1)

Also

Area of ΔORS = 1/2 × OS × RN

= 1/2 × 5 × RN …(2)

From equation (1) and (2)

(1/2) × 5 × RN = 12

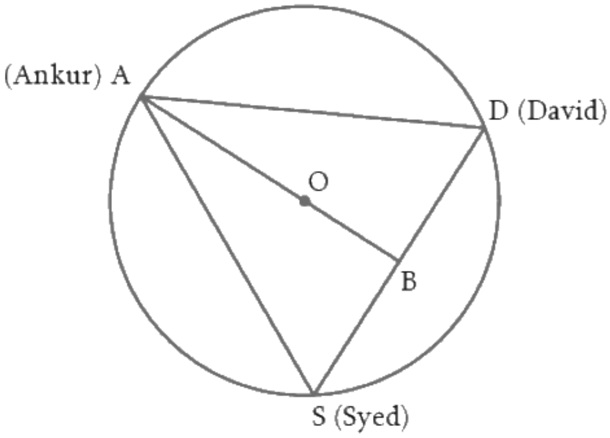
RN = 24/5 = 4.8m

RM = 2 × RN = 2 × 4.8 = 9.6 m

Distance between Reshma and Salma is 9.6 m.

6. A circular park of radius 20m is situated in a colony. Three boys, Ankur, Syed and David, are sitting at equal distances on its boundary, each having a toy telephone in his hands to talk to each other. Find the length of the string of each phone.

Answer: Centre and Centroid are the same for an equilateral triangle, and it divides the median in the ratio 2 : 1.



Let A, D, S denote the positions of Ankur, David, and Syed, respectively.

ΔADS is an equilateral triangle since all the 3 boys are equidistant from one another.

Let B denote the mid-point of DS, and hence AB is the median and perpendicular bisector of DS.

Hence ΔABS is a right-angled triangle with ∠ABS = 90º.

O (centroid) divides the line AB in the ratio 2 : 1. So OA : OB = 2 : 1.

OA/OB = 2/1

Since OA = 20 (given)

thus, OB = 10m

AB = OA + OB = 20 + 10 = 30 m ….(1)

Let the side of the equilateral triangle ΔADS be 2x.

AD = DS = SA = 2x …. (2)

Since B is the mid-point of DS, we get BS = BD = x …. (3)

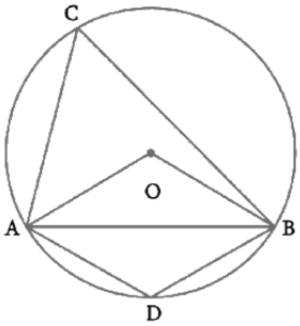
Applying Pythagoras theorem to ΔABD, we get:

$AD^2 = AB^2 + BD^2$

$(2x)^2 = 30^2 + x^2$

$4x^2 = 900 + x^2$

$3x^2 = 900$



$x^2 = 300$

x = 10√3

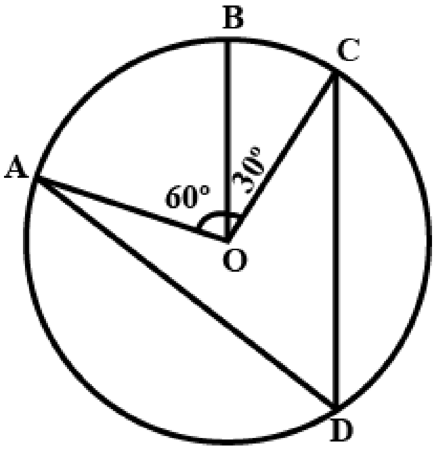
x = 17.32

AD = DS = SA = 2x = 34.64 m

Length of the string = Distance between them = AD or DS or SA = 34.64 m.

Exercise 10.5:

1. **In Fig. 10.36, A, B and C are three points on a circle with centre O such that ∠BOC = 30º and ∠AOB = 60º. If D is a point on the circle other than the arc ABC, find ∠ADC.**



Answer: The angle subtended by an arc at the center is double the angle subtended by it at any point on the remaining part of the circle.

∠AOC = ∠AOB + ∠BOC = 90°

∠AOC = 2 ∠ADC (By Theorem 10.8)

∠ADC = 1/2 ∠AOC

∠ADC = 1/2 × 90 = 45°

∴ ∠ADC = 45°

2. A chord of a circle is equal to the radius of the circle. Find the angle subtended by the chord at a point on the minor arc and also at a point on the major arc

Answer: The angle subtended by an arc at the center is double the angle subtended by it at any point on the remaining part of the circle.

A quadrilateral ABCD is called cyclic if all the four vertices of it lie on a circle and the sum of either pair of opposite angles of a cyclic quadrilateral is 180°.

Draw a circle with any radius and center O. Let AO and BO be the 2 radii of the circle and let AB be the chord equal to the length of the radius. Join them to form a triangle.

Here OA = OB = AB

Hence ΔABO becomes an equilateral triangle.

Draw 2 points C and D on the circle such that they lie on the major arc and minor arc, respectively.

Since ΔABO is an equilateral triangle, we get ∠AOB = 60°.

For the arc AB, ∠AOB = 2∠ACB as we know that the angle subtended by an arc at the center is double the angle subtended by it at any point on the remaining part of the circle.

∠ACB = 1/2 ∠AOB = 1/2 × 60 = 30°

As you can notice the points A, B, C, and D lie on the circle. Hence ABCD is a cyclic quadrilateral.

We know that the sum of either pair of opposite angles of a cyclic quadrilateral is 180°.

Therefore,

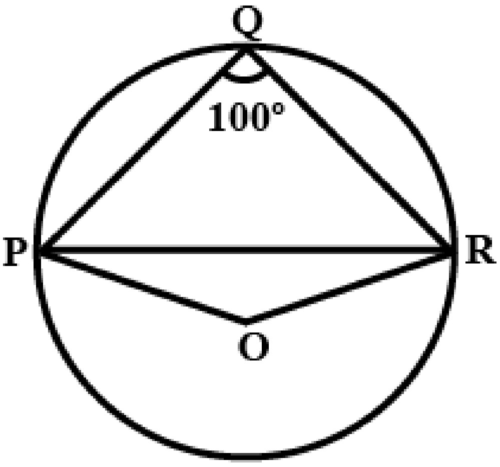
∠ACB + ∠ADB = 180°

30° + ∠ADB = 180°

∠ADB = 150°

So, when the chord of a circle is equal to the radius of the circle, the angle subtended by the chord at a point on the minor arc is 150° and also at a point on the major arc is 30°.

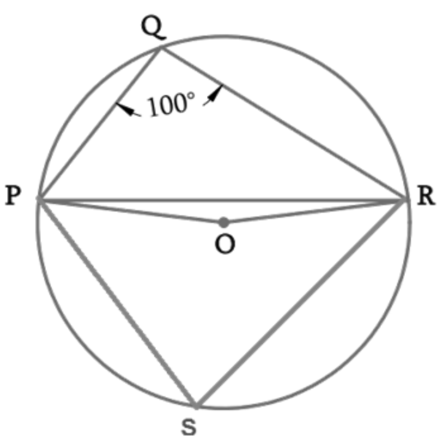
3. In Fig.10.37, ∠PQR = 100° where P, Q and R are points on a circle with centre O. Find ∠OPR



Answer: The angle subtended by an arc at the center is double the angle subtended by it at any point on the remaining part of the circle.

A quadrilateral ABCD is called cyclic if all the four vertices of it lie on a circle and the sum of either pair of opposite angles of a cyclic quadrilateral is 180°.

Mark any point on the major arc side (opposite side to point Q) as S.



Since all points P, Q, R, S lie on the circle, PQRS becomes a cyclic quadrilateral.

We know that the sum of either pair of opposite angles of a cyclic quadrilateral is 180°.

Therefore,

∠PQR + ∠PSR = 180°

100° + ∠PSR = 180°

∠PSR = 180° - 100° = 80°

We know that the angle subtended by an arc at the center is double the angle subtended by it at any point on the remaining part of the circle.

Therefore,

∠POR = 2∠PSR

= 2 × 80°

= 160°

Consider the ΔOPR. It is an isosceles triangle as OP = OR = Radius of the circle.

Thus, ∠OPR = ∠ORP

Sum of all angles in a triangle is 180°.

Therefore,

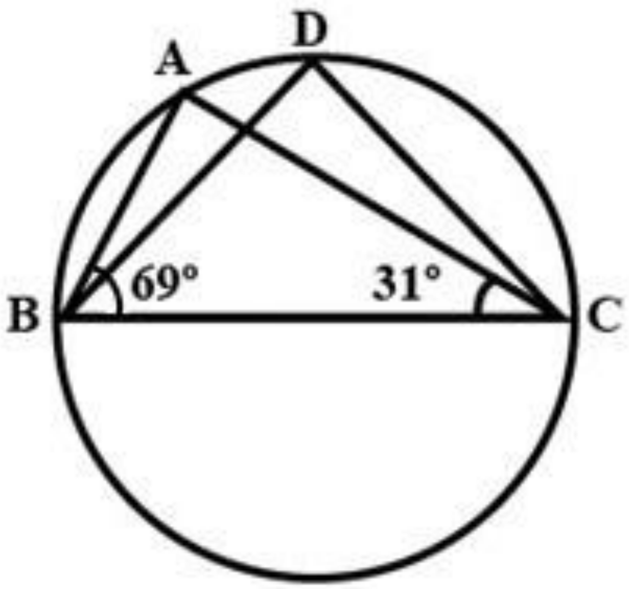
∠OPR + ∠POR + ∠ORP = 180°

∠OPR + 160° + ∠OPR = 180°

2∠OPR = 180° - 160°

∠OPR = 10°

4. In Fig. 10.38, ∠ABC = 69° and ∠ACB= 31°, find ∠BDC.



Answer: Concepts that we will use to find the required answer:

- The sum of angles in a triangle is 180°.
- Angles in the same segment are equal.

Consider the ΔABC, the sum of all angles will be 180°.

∠ABC + ∠BAC + ∠ACB = 180°

69° + ∠BAC + 31° = 180°

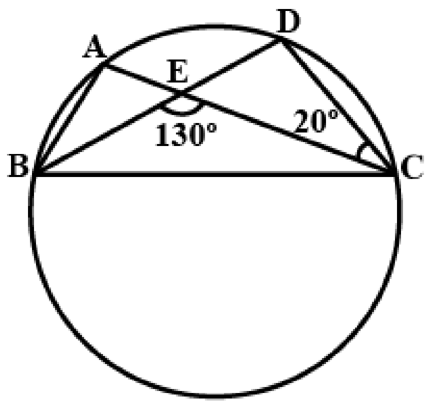
∠BAC = 180° -100°

= 80°

We know that angles in the same segment of a circle are equal.

So, ∠BDC = ∠BAC = 80°

5. In Fig. 10.39, A, B, C and D are four points on a circle. AC and BD intersect at a point E such that ∠BEC = 130° and ∠ECD = 20°. Find ∠BAC



Answer:

We will use the following concepts to answer the question.

• The sum of angles in a triangle is 180°.

• Angles in the same segment are equal.

Consider the straight-line BD. As the line AC intersects with the line BD, the sum of two adjacent angles so formed is 180°.

Therefore, ∠BEC + ∠DEC = 180°

130° + ∠DEC = 180°

∠DEC =180° - 130° = 50°

Consider the ΔDEC, the sum of all angles will be 180º.

∠DEC + ∠EDC + ∠ECD = 180°

50° + ∠EDC + 20° = 180°

∠EDC = 180° - 70° = 110°

∴ ∠BDC = ∠EDC = 110°

We know that angles in the same segment of a circle are equal.

∴ ∠BAC = ∠BDC = 110°

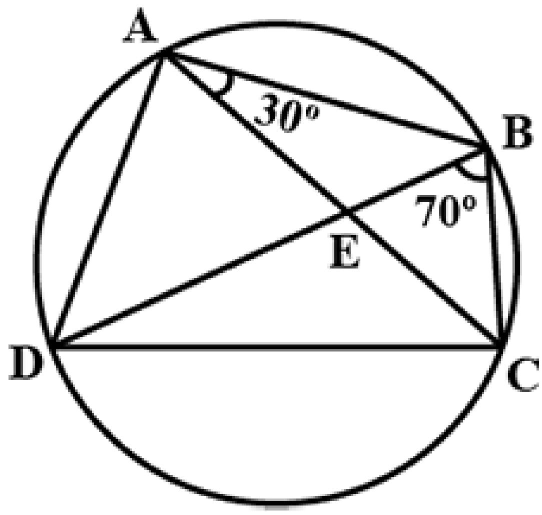
6. ABCD is a cyclic quadrilateral whose diagonals intersect at a point E. If ∠DBC = 70°, ∠BAC is 30°, find ∠BCD. Further, if AB = BC, find ∠ECD

Answer:

We will use the following concepts to answer the question:

- A quadrilateral ABCD is called cyclic if all the four vertices of it lie on a circle.
- The sum of either pair of opposite angles of a cyclic quadrilateral is 180°.
- The sum of angles in a triangle is 180°.
- Angles in the same segment are equal.

Based on the data given, let's draw the figure as shown below.



In the triangles, ABD and BCD, ∠CAD = ∠CBD = 70°. (Angles in the same segment are equal)

Hence, ∠BAD = ∠CAB + ∠DAC

= 30° + 70° = 100°

Thus, ∠BAD = 100°

Since ABCD is a cyclic quadrilateral, the sum of either pair of opposite angles of a cyclic quadrilateral is 180º.

∠BAD + ∠BCD = 180°

∠BCD = 180° - 100°

= 80°

Thus, ∠BCD = 80°

Also given AB = BC.

So, ∠BCA = ∠BAC = 30° (Base angles of isosceles triangle are equal)

∠ECD = ∠BCD - ∠BCA

= 80° - 30°

= 50°

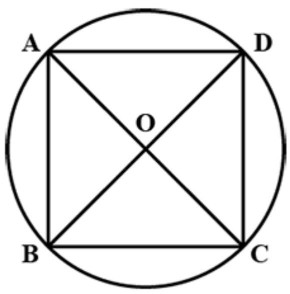
Thus, ∠ECD = 50°

7. If diagonals of a cyclic quadrilateral are diameters of the circle through the vertices of the quadrilateral, prove that it is a rectangle.

Answer:

Concepts used to solve the question:

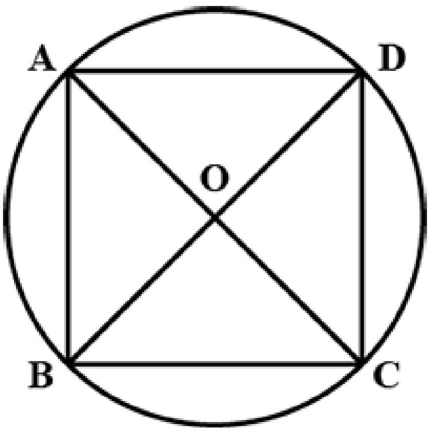
- The angle subtended by an arc at the center is double the angle subtended by it at any point on the remaining part of the circle.
- The sum of either pair of opposite angles of a cyclic quadrilateral is 180°.
- Diameter is the longest chord.



Let BD be the diameter of the circle, which is also a chord. Then, ∠BOD = 180°

Since AC and BD are diameters of the circle

⇒ AC = BD



We know that the angle subtended by an arc at the center is double the angle subtended by it at any point on the remaining part of the circle.

∴ ∠BAD = 1/2 × ∠BOD = 90°

Similarly, ∠BCD = 90°

Now, considering AC as the diameter of the circle, we get ∠AOC = 180°

We know that the angle subtended by an arc at the center is double the angle subtended by it at any point on the remaining part of the circle.

∠ABC = 1/2 × ∠AOC = 90°

Similarly, ∠ADC = 90°

Let us now consider the triangles ΔABC and ΔBAD,

∠ABC = ∠BAD [Each equal to 90°]

AB = BA [Common side of triangle ABC and BAD]

AC = BD [Diameter of the circle]

∴ ΔABC ≅ ΔBAD [By RHS congruence criteria]

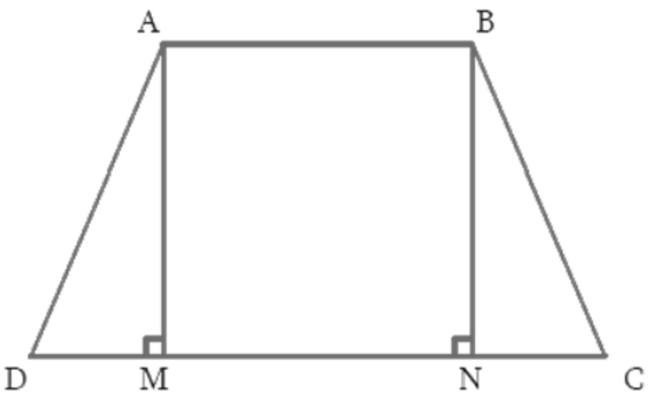
⇒ BC = AD [By Corresponding parts of Congruent triangles theorem]

Similarly, AB = DC

As you can see, all the angles at the corners are 90º and the opposite sides are equal. We can say that the shape joining the vertices ABCD is a rectangle.

8. If the non-parallel sides of a trapezium are equal, prove that it is cyclic

Answer:



We know that, if the sum of a pair of opposite angles of a quadrilateral is 180°, the quadrilateral is cyclic.

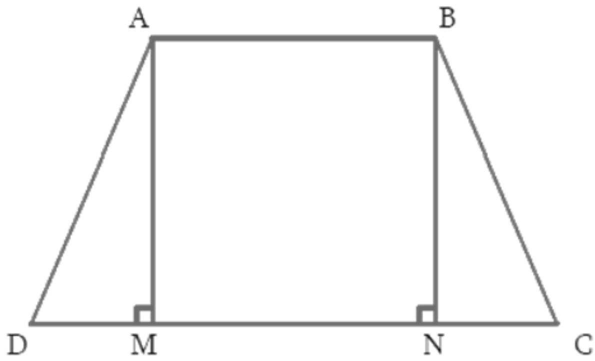
Draw a trapezium ABCD with AB || CD

AD and BC are the non-parallel sides that are equal. AD = BC. Draw AM ⊥ CD and BN ⊥ CD.

Consider ΔAMD and ΔBNC

AD = BC (Given)

∠AMD = ∠BNC (90°)



AM = BN (Perpendicular distance between two parallel lines is same)

By RHS congruence, ΔAMD ≅ ΔBNC.

Using CPCT, ∠ADC = ∠BCD.....(1)

∠BAD and ∠ADC are on the same side of transversal AD.

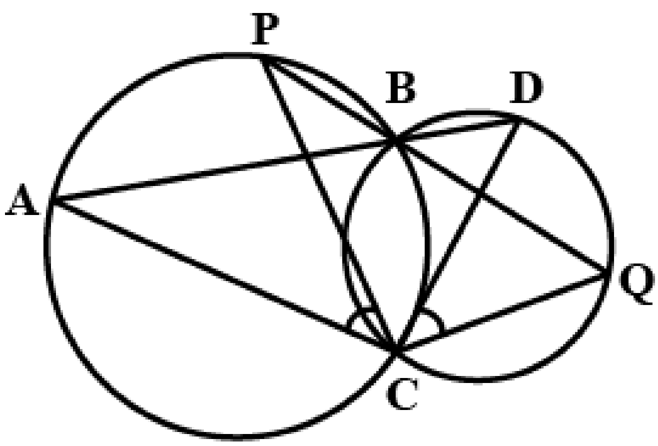
∠BAD + ∠ADC = 180°

∠BAD + ∠BCD = 180° [From equation(1)]

This equation proves that the sum of opposite angles is supplementary. Hence, ABCD is a cyclic quadrilateral.

9. Two circles intersect at two points B and C. Through B, two line segments ABD and PBQ are drawn to intersect the circles at A, D, P and Q respectively (see Fig. 10.40). Prove that ∠ACP = ∠QCD.

Answer:



∠ACP and ∠ABP lie in the same segment. Similarly, ∠DCQ and ∠DBQ lie in the same segment.

We know that angles in the same segment of a circle are equal.

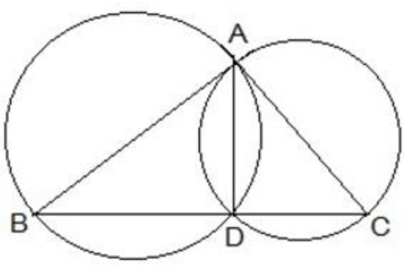
So, we get ∠ACP = ∠ABP and ∠QCD = ∠QBD

Also, ∠QBD = ∠ABP (Vertically opposite angles)

Therefore, ∠ACP = ∠QCD.

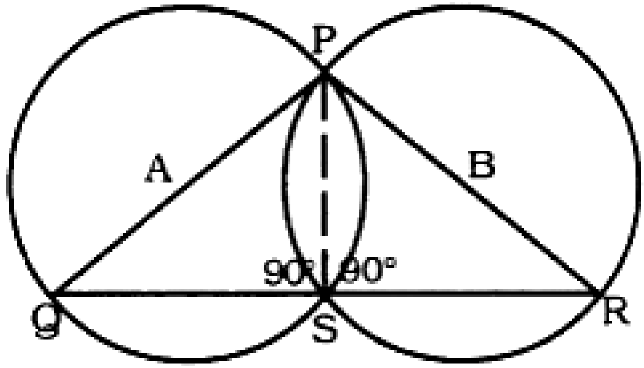
10. If circles are drawn taking two sides of a triangle as diameters, prove that the point of intersection of these circles lie on the third side.

Answer:



We know that an angle in a semicircle is a right angle. By using this fact, we can show that BDC is a line that will lead to proving that the point of intersection lies on the third side.

Since angle in a semicircle is a right angle, we get:



∠ADB = 90° and ∠ADC = 90°

∠ADB + ∠ADC = 90° + 90°

⇒ ∠ADB + ∠ADC = 180°

⇒ BDC is a straight line.

D lies on BC

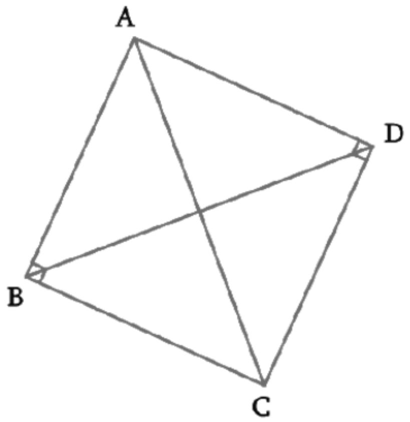
Hence, the point of intersection of circles lies on the third side BC.

11. ABC and ADC are two right triangles with common hypotenuse AC. Prove that ∠CAD = ∠CBD

Answer:

We know that, the sum of all angles in a triangle is 180°.

If the sum of pair of opposite angles in a quadrilateral is 180°, then it is a cyclic quadrilateral.



Consider ΔABC,

∠ABC + ∠BCA + ∠CAB = 180° (Angle sum property of a triangle)

90° + ∠BCA + ∠CAB = 180°

∠BCA + ∠CAB = 90°.....(1)

Consider ΔADC,

∠CDA + ∠ACD + ∠DAC = 180° (Angle sum property of a triangle)

90° + ∠ACD + ∠DAC = 180°

∠ACD + ∠DAC = 90°.....(2)

Adding Equations (1) and (2), we obtain

∠BCA + ∠CAB + ∠ACD + ∠DAC = 180°

(∠BCA +∠ACD) + (∠CAB + ∠DAC) = 180°

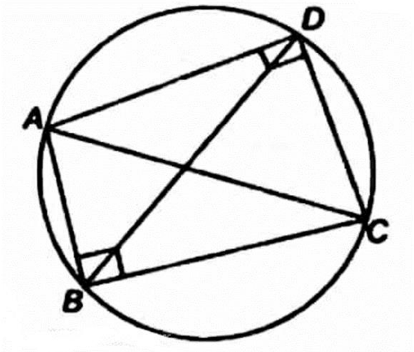
∠BCD + ∠DAB = 180°.....(3)

However, it is given that

∠B + ∠D = 90° + 90° = 180°.....(4)

From Equations (3) and (4), it can be observed that the sum of the measures of opposite angles of quadrilateral ABCD is 180°. Therefore, it is a cyclic quadrilateral.

Since it is a cylclic quadrilateral the below figure can be drawn.



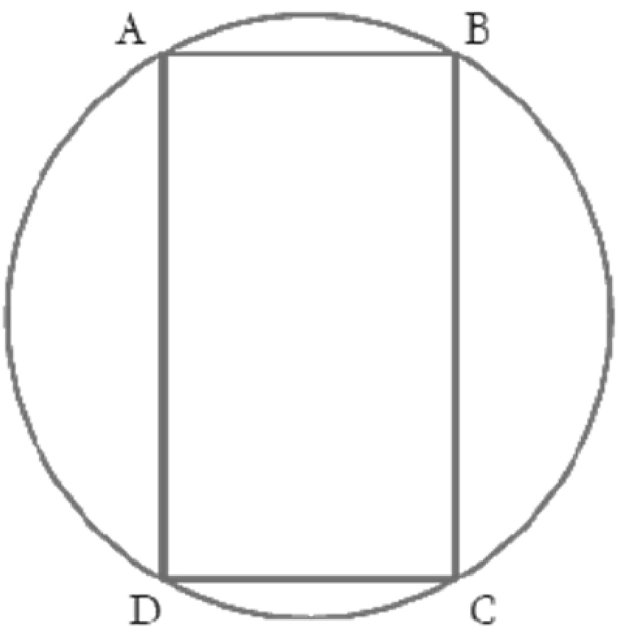
Consider chord CD. ∠CAD and ∠CBD are formed on the same segment CD.

∠CAD = ∠CBD (Angles in the same segment are equal)

12. Prove that a cyclic parallelogram is a rectangle

Answer:

The sum of either pair of opposite angles of a cyclic quadrilateral is 180°. Using this fact, we can show each angle of a cyclic parallelogram as 90°, proving the statement it is a rectangle.



Let ABCD be the cyclic parallelogram.

We know that opposite angles of a parallelogram are equal.

∠A = ∠C and ∠B = ∠D ... (1)

We know that the sum of either pair of opposite angles of a cyclic quadrilateral is 180°.

∠A + ∠C = 180°

∠A + ∠A = 180° (From equation (1))

2∠A = 180°

∠A = 90°

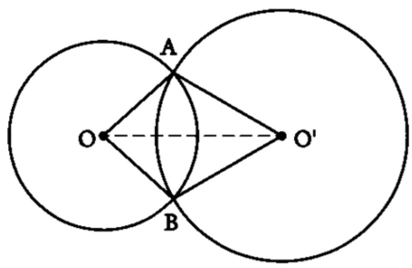
We know that if one of the interior angles of a parallelogram is 90°, all the other angles will also be equal to 90°.

Since all the angles in the parallelogram are 90°, we can say that parallelogram ABCD is a rectangle.

Exercise 10.6:

1. Prove that the line of centres of two intersecting circles subtends equal angles at the two points of intersection

Answer: Draw two intersecting circles with centers O and O', respectively. Join these two centers. Let the points of intersection be A and B.



We need to prove that ∠OAO'= ∠OBO'

Consider ΔOAO' and ΔOBO'

OA = OB (Radii of a circle with center O)

O'A = O'B (Radii of a circle with center O')

OO'= OO' (Common)

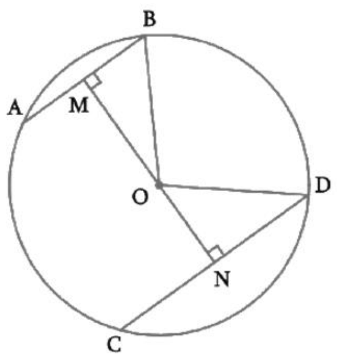
Therefore, by SSS criteria, ΔOAO' and ΔOBO' are congruent to each other.

By CPCT, ∠OAO'= ∠OBO'

Hence it is proved that the line of centers of two intersecting circles subtends equal angles at the two points of intersection.

2. Two chords AB and CD of lengths 5 cm and 11 cm respectively of a circle are parallel to each other and are on opposite sides of its centre. If the distance between AB and CD is 6 cm, find the radius of the circle

Answer:



Draw two parallel chords AB and CD of lengths 5 cm and 11 cm. Let the center of the circle be O. Join one end of each chord to the center.

Draw two perpendiculars OM and ON to AB and CD, respectively, which bisects the chords.

Thus, MB = 2.5 cm and ND = 5.5 cm [The perpendicular drawn from the center of the circle to the chords bisects it.]

Let OM = x and ON = 6 - x

Consider ΔOMB

By Pythagoras theorem,

$OM^2 + MB^2 = OB^2$

$x^2 + 2.5^2 = OB^2$

$x^2 + 6.25 = OB^2$..................(1)

Consider ΔOND

By Pythagoras theorem,

$ON^2 + ND^2 = OD^2$

$(6 - x)^2 + 5.5^2 = OD^2$

$36 + x^2 - 12x + 30.25 = OD^2$

$x^2 - 12x + 66.25 = OD^2$............... (2)

OB and OD are the radii of the circle. Therefore OB = OD.

Thus, $OB^2 = OD^2$

Equating (1) and (2) we get,

$x^2 + 6.25 = x^2 - 12x + 66.25$

$12x = 60$

$x = 5$

Substituting the value of x in (1),

$OB^2 = x^2 + 6.25$

$OB^2 = 5^2 + 6.25$

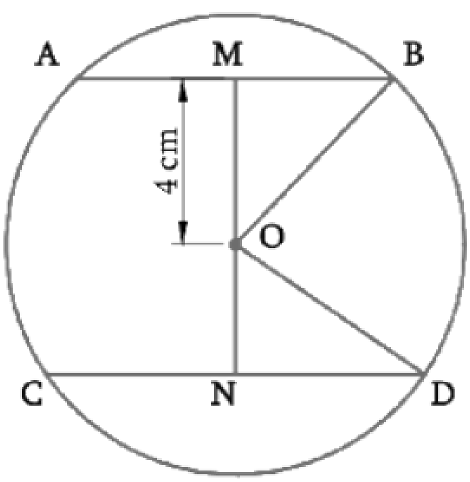
$OB^2 = 31.25$

OB = 5.59 (approx.)

Thus, we get the radius of the circle = 5.59 cm.

3. The lengths of two parallel chords of a circle are 6 cm and 8 cm. If the smaller chord is at distance 4 cm from the centre, what is the distance of the other chord from the centre?

Answer:



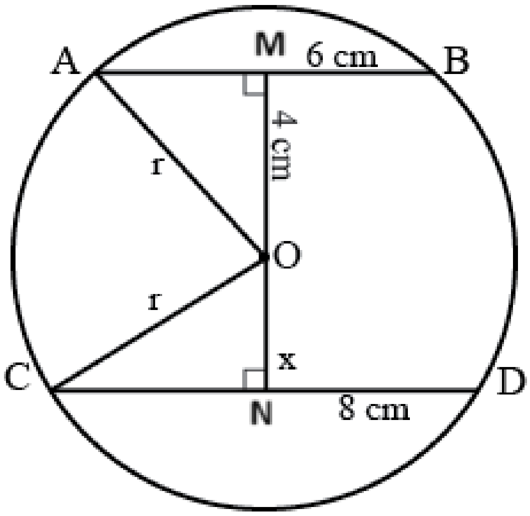
The perpendicular drawn from the center of the circle to the chords bisects it.

Draw two parallel chords AB and CD of lengths 6 cm and 8 cm. Let the circle's center be O. Join one end of each chord to the center. Draw 2 perpendiculars OM and ON to AB and CD, respectively, which bisects the chords.

AB = 6 cm CD = 8 cm MB = 3 cm ND = 4 cm

Given OM = 4 cm and let ON = x cm

Consider ΔOMB



By Pythagoras theorem,

$OM^2 + MB^2 = OB^2$

$4^2 + 3^2 = OB^2$

$OB^2 = 25$

OB = 5 cm

OB and OD are the radii of the circle.

Therefore OD = OB = 5 cm.

Consider ΔOND

By Pythagoras theorem,

$ON^2 + ND^2 = OD^2$

$x^2 + 4^2 = 5^2$

$x^2 = 25 - 16$

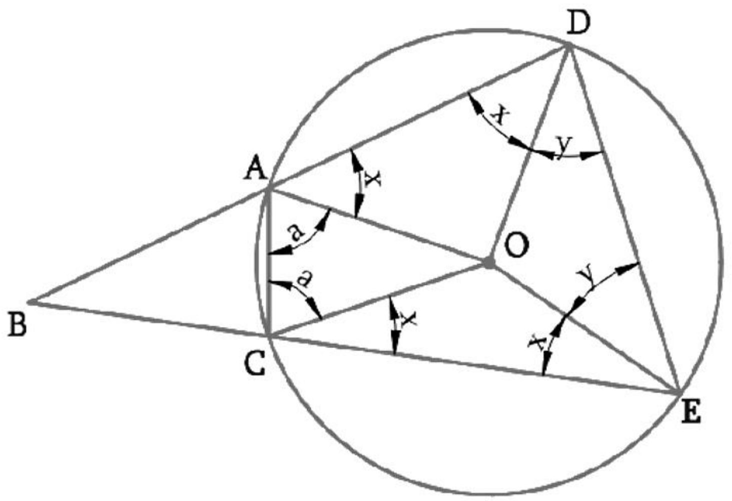
$x^2 = 9$

x = 3

The distance of the chord CD from the center is 3 cm.

4. Let the vertex of an angle ABC be located outside a circle and let the sides of the angle intersect equal chords AD and CE with the circle. Prove that ∠ABC is equal to half the difference of the angles subtended by the chords AC and DE at the centre.

Answer: Let's construct a diagram based on the given question as shown below:



To prove: ∠ABC = 1/2 (∠DOE - ∠AOC)

Consider ΔAOD and ΔCOE,

OA = OC (Radii of the circle)

OD = OE (Radii of the circle)

AD = CE (Given)

Thus, ΔAOD ≅ ΔCOE (SSS Congruence Rule)

∠OAD = ∠OCE (By CPCT) ... (1)

∠ODA = ∠OEC (By CPCT) ... (2)

Also,

∠OAD = ∠ODA (As OA = OD) ... (3)

From Equations (1), (2), and (3), we obtain

∠OAD = ∠OCE = ∠ODA = ∠OEC

Let ∠OAD = ∠OCE = ∠ODA = ∠OEC = x

In ΔOAC,

OA = OC

∴ ∠OCA = ∠OAC (Angle a)

In ΔODE,

OD = OE

∠OED = ∠ODE (Angle y)

ADEC is a cyclic quadrilateral.

∴ ∠CAD + ∠DEC = 180° (Opposite angles are supplementary)

x + a + x + y = 180°

2x + a + y = 180°

y = 180° - 2x - a ... (4)

However, ∠DOE = 180° - 2y and, ∠AOC = 180° − 2a

∠DOE - ∠AOC = 2a - 2y

= 2a - 2 (180° - 2x - a) [From equation (4)]

= 4a + 4x - 360° ... (5)

∠BAC + ∠CAD = 180° (Linear pair)

∴ ∠BAC = 180º - ∠CAD = 180° - (a + x) ... (6)

Similarly, ∠ACB = 180° - (a + x) ... (7)

In ΔABC,

∠ABC + ∠BAC + ∠ACB = 180° (Angle sum property of a triangle)

∠ABC = 180° - ∠BAC - ∠ACB

= 180° - (180° - a - x) - (180° - a - x) [From (6) and (7)]

= 2a + 2x -180°

= 1/2 [4a + 4x - 360°]

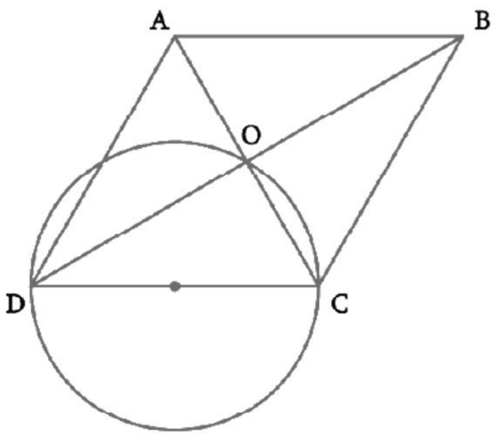
Using Equation (5)

∠ABC = 1/2 (∠DOE - ∠AOC)

Hence it is proved that ∠ABC is equal to half the difference of the angles subtended by the chords AC and DE at the center.

5. Prove that the circle drawn with any side of a rhombus as diameter, passes through the point of intersection of its diagonals.

Answer: Let's draw a figure according to the given question.



Let ABCD be a rhombus in which diagonals intersect at point O, and a circle is drawn by taking side CD as its diameter. We know that a diameter subtends 90° on the arc.

Therefore, ∠COD = 90°

Also, in the rhombus, the diagonals intersect each other at 90°.

∠AOB = ∠BOC = ∠COD = ∠DOA = 90°

But ∠COD is 90° and this can only happen on a semicircle with diameter DC since the angle subtended by the diameter on a semicircle is 90°.

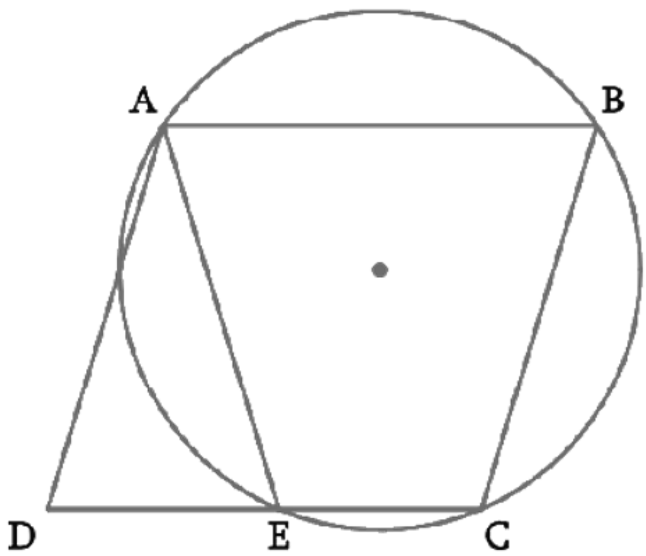
Clearly, point O has to lie on the circle.

Thus, the circle passes through the point of intersection of its diagonals O.

6. ABCD is a parallelogram. The circle through A, B and C intersect CD (produced if necessary) at E. Prove that AE = AD.

Answer: A quadrilateral ABCD is called cyclic if all the four vertices of the quadrilateral lie on a circle.

The sum of either pair of opposite angles of a cyclic quadrilateral is 180^0.



We can see that ABCE is a cyclic quadrilateral.

We know that in a cyclic quadrilateral, the sum of the opposite angles is 180°.

∠AEC + ∠CBA = 180°

∠AEC + ∠AED = 180° (Linear pair)

Thus, ∠AED = ∠CBA...................(1)

We know that in a parallelogram, opposite angles are equal.

∠ADE = ∠CBA.................. (2)

From (1) and (2),

∠AED = ∠ADE

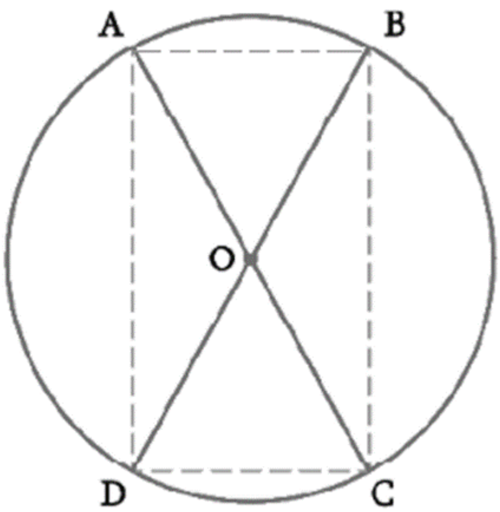
Therefore, AD = AE (sides opposite to equal angles in a triangle are equal).

Hence proved.

7. AC and BD are chords of a circle which bisect each other. Prove that (i) AC and BD are diameters, (ii) ABCD is a rectangle

Answer: A quadrilateral ABCD is called cyclic if all the four vertices of it lie on a circle and the sum of either pair of opposite angles of a cyclic quadrilateral is 180º.

Let's construct a figure according to the given question.



Let AC and BD be two chords intersecting at O.

In ΔAOB and ΔCOD,

OA = OC (Given)

OB = OD (Given)

∠AOB = ∠COD (Vertically opposite angles)

Hence, ΔAOB ≅ ΔCOD (SAS congruence rule)

AB = CD (By CPCT)

Similarly, it can be proved that ΔAOD ≅ ΔCOB

Hence, AD = CB (By CPCT)

Since in quadrilateral ABCD, opposite sides are equal in length, ABCD is a parallelogram.

We know that opposite angles of a parallelogram are equal.

Therefore, ∠A = ∠C

However, ∠A + ∠C = 180° (ABCD is a cyclic quadrilateral)

∠A + ∠A = 180°

2∠A = 180°

∴ ∠A = 90°

ABCD is a parallelogram and one of its interior angles is 90°, therefore, it is a rectangle.

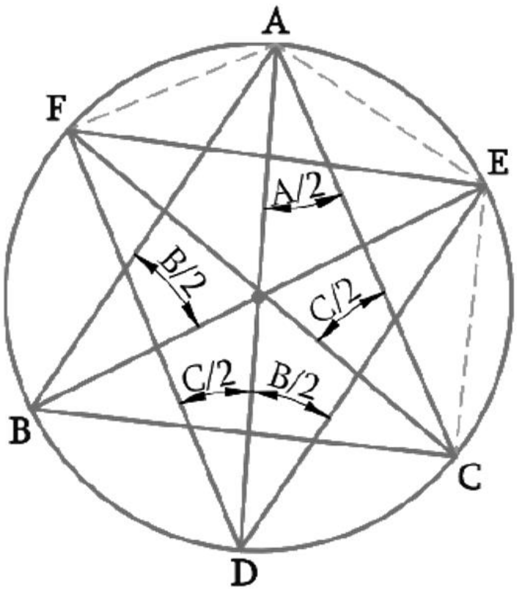
∠A is the angle subtended by chord BD, ∠A = 90°, therefore, BD should be the diameter of the circle [Since, angle in a semicircle is a right angle]

Similarly, AC is the diameter of the circle.

Thus, (i) AC and BD are diameters, and (ii) ABCD is a rectangle, proved

8. Bisectors of angles A, B and C of a triangle ABC intersect its circumcircle at D, E and F respectively. Prove that the angles of the triangle DEF are 90° - 1/2 A, 90° - 1/2 B, 90° - 1/2 C

Answer: We know that angles in the same segment are equal.



A diagram is constructed as per the given question.

It is given that BE is the bisector of ∠B, AD is the bisector of ∠A and CF is the bisector of ∠C.

Thus, ∠ABE = ∠B/2

However, ∠ADE = ∠ABE (Angles in the same segment for chord AE)

Thus, ∠ADE = ∠B/2

Similarly, ∠ADF = ∠ACF = ∠C/2 (Angle in the same segment for chord AF)

∠D = ∠ADE + ∠ADF

= ∠B/2 + ∠C/2 [Since ∠ADE = ∠B/2 and ∠ADF = ∠C/2]

= 1/2 (∠B + ∠C)

= 1/2 (180° - ∠A) [Angle sum property of triangle ABC]

= 90° - 1/2 A

Similarly, it can be proved for

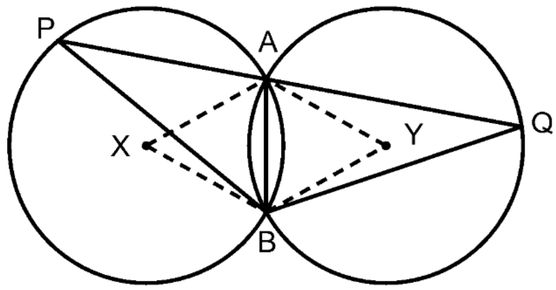
∠E = 90° - 1/2 B

∠F = 90° - 1/2 C.

Thus we have proved that the angles of the triangle DEF are 90° - 1/2 A, 90° - 1/2 B, 90° - 1/2 C.

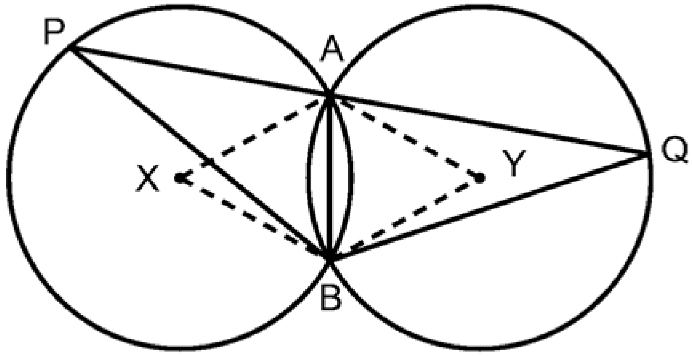
9. Two congruent circles intersect each other at points A and B. Through A any line segment PAQ is drawn so that P, Q lie on the two circles. Prove that BP = BQ.

Answer: Let's represent a diagram according to the given question.



AB is the common chord to both circles.

Since the circles are congruent, their radii are equal.



In triangles ABX and ABY

AB = AB (Common)

AX = AY (equal radii)

BX = BY (equal radii)

By SSS congruence criteria, triangles ABX and ABY are congruent.

Hence ∠X = ∠Y [CPCT] …(1)

Now, we know that the angle subtended by an arc at the center is double the angle subtended by it at any point on the remaining part of the circle

Thus,

∠APB = 1/2 ∠X …(2)

∠AQB = 1/2 ∠Y …(3)

Therefore,

∠APB = ∠AQB [From equations (1), (2) and (3)]

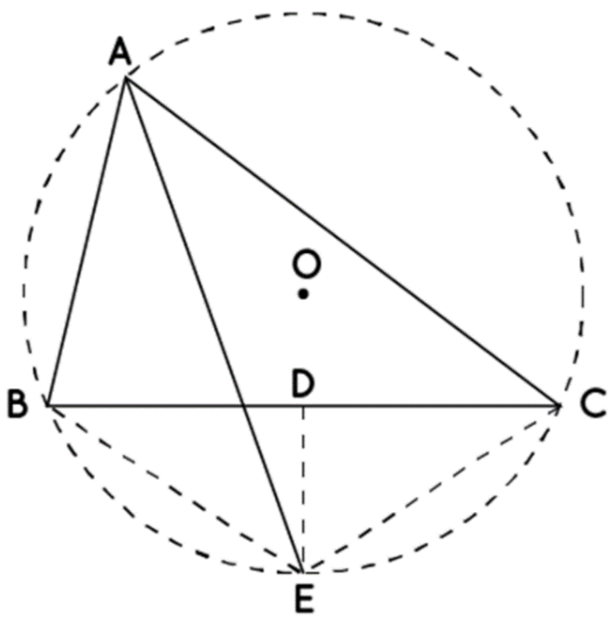
Consider the ΔBPQ,

∠APB = ∠AQB

This implies that ΔBPQ is an isosceles triangle as base angles are equal.

Therefore, we get BP = BQ, sides opposite to equal sides in a triangle are equal.

10. In any triangle ABC, if the angle bisector of ∠A and perpendicular bisector of BC intersect, prove that they intersect on the circumcircle of the triangle ABC.



Answer: Let's represent a diagram according to the given question.

Let AE be the angle bisector of ∠A.

We need to prove that ED is the perpendicular bisector of BC.

∠BAE = ∠CAE .... (1) [Since, AE is the angle bisector of ∠A]

Now, ∠EBC = ∠CAE .... (2) [Angles subtended by the same arc EC]

Also, ∠ECB = ∠BAE .... (3) [Angles subtended by the same arc BE]

But we know that, ∠BAE = ∠CAE [From equation (1)]

Hence, ∠EBC = ∠ECB [From equations (2) and (3)]

Therefore, BE = EC [Sides opposite to equal angles are equal]

Thus, point E is equidistant from the points B and C. This is only possible when E lies on the perpendicular bisector of BC.

Thus, ED is the perpendicular bisector of BC.

Therefore, the perpendicular bisector of side BC and the angle bisector of ∠A meet on the circumcircle of triangle ABC at point E.