Exercise 7.1

1. Which of the following numbers are not perfect cubes?

(i) 216 (ii) 128 (iii) 1000 (iv) 100 (v) 46656

Answer: Cube root is the number that needs to be multiplied three times to get the original number.

For a number to be a perfect cube, all its prime factors should exist in triplets.

(i) 216

2	216
2	108
2	54
3	27
3	9
	3

$216 = \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3}$

$= 2^3 \times 3^3$

∴ 216 is a perfect cube

(ii) 128

2	128
2	64
2	32
2	16
2	8
2	4
	2

$128 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times 2$

$= 2^3 \times 2^3 \times 2$

∴ 128 is not a perfect cube

(iii) 1000

2	1000
2	500
2	250
5	125
5	25
5	5
	1

$1000 = \underline{2 \times 2 \times 2} \times \underline{5 \times 5 \times 5}$

$= 2^3 \times 5^3$

∴ 1000 is a perfect cube

(iv) 100

2	100
2	50
5	25
	5

$100 = \underline{2 \times 2} \times 5 \underline{\times 5}$

$= 2^2 \times 5^2$

∴ 100 is not a perfect cube

(v) 46656

$46656 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3} \times \underline{3 \times 3 \times 3}$

$= 2^3 \times 2^3 \times 3^3 \times 3^3$

∴ 46656 is a perfect cube

2	46656
2	23328
2	11664
2	5832
2	2916
2	1458
3	729
3	243
3	81
3	27
3	9
	3

2. Find the smallest number by which each of the following numbers must be multiplied to obtain a perfect cube. (i) 243 (ii) 256 (iii) 72 (iv) 675 (v) 100

Answer: A number is a perfect cube only when each factor in the prime factorization of the given number exists in triplets. Using this concept, the smallest number can be identified.

(i) 243

$243 = \underline{3 \times 3 \times 3} \times 3 \times 3$

$= 3^3 \times 3^2$

Here, one group of 3's is not existing as a triplet. To make it a triplet, we need to multiply by 3.

3	243
3	81
3	27
3	9
	3

Thus, $243 \times 3 = \underline{3 \times 3 \times 3} \times \underline{3 \times 3 \times 3} = 729$ is a perfect cube

Hence, the smallest natural number by which 243 should be multiplied to make a perfect cube is 3.

(ii) 256

$256 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times 2 \times 2$

$= 2^3 \times 2^3 \times 2 \times 2$

Here, one of the groups of 2's is not a triplet. To make it a triplet, we need to multiply by 2.

2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

Thus, $256 \times 2 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} = 512$ is a perfect cube

Hence, the smallest natural number by which 256 should be multiplied to make a perfect cube is 2.

(iii) 72

$72 = \underline{2 \times 2 \times 2} \times 3 \times 3$

$= 2^3 \times 3^2$

2	72
2	36
2	18
3	9
	3

Here, the group of 3's is not a triplet. To make it a triplet, we need to multiply by 3.

Thus, $72 \times 3 = \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3} = 216$ is a perfect cube

Hence, the smallest natural number by which 72 should be multiplied to make a perfect cube is 3.

(iv) 675

$675 = \underline{5 \times 5} \times \underline{3 \times 3 \times 3}$

$= 5^2 \times 3^3$

5	675
5	135
3	27
3	9
	3

Here, the group of 5's is not a triplet. To make it a triplet, we need to multiply by 5.

Thus, $675 \times 5 = \underline{5 \times 5 \times 5} \times \underline{3 \times 3 \times 3} = 3375$ is a perfect cube

Hence, the smallest natural number by which 675 should be multiplied to make a perfect cube is 5.

(v) 100

$100 = \underline{2 \times 2} \times \underline{5 \times 5}$

$= 2^2 \times 5^2$

2	100
2	50
5	25
	5

Here both the prime factors are not triplets. To make them triplets, we need to multiply by one 2 and one 5.

Thus, $100 \times 2 \times 5 = \underline{2 \times 2 \times 2} \times \underline{5 \times 5 \times 5} = 1000$ is a perfect cube

Hence, the smallest natural number by which 100 should be multiplied to make a perfect cube is $2 \times 5 = 10$

3. Find the smallest number by which each of the following numbers must be divided to obtain a perfect cube. (i) 81 (ii) 128 (iii) 135 (iv) 192 (v) 704

Answer: A number is a perfect cube only when each factor in the prime factorization is grouped in triples. Using this concept, the smallest number can be identified.

(i) 81

$81 = \underline{3 \times 3 \times 3} \times 3$

$= 3^3 \times 3$

3	81
3	27
3	9
	3

Here, the prime factor 3 is not grouped as a triplet. Hence, we divide 81 by 3, so that the obtained number becomes a perfect cube.

Thus, $81 \div 3 = 27 = 3^3$ is a perfect cube.

Hence the smallest number by which 81 should be divided to make a perfect cube is 3.

(ii) 128

$128 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times 2$

$= 2^3 \times 2^3 \times 2$

Here, the prime factor 2 is not grouped as a triplet. Hence, we divide 128 by 2, so that the obtained number becomes a perfect cube.

Thus, $128 \div 2 = 64 = 4^3$ is a perfect cube.

Hence the smallest number by which 128 should be divided to make a perfect cube is 2.

2	128
2	64
2	32
2	16
2	8
2	4
	2

(iii) 135

$135 = \underline{3 \times 3 \times 3} \times 5$

$= 3^3 \times 5$

Here, the prime factor 5 is not a triplet. Hence, we divide 135 by 5, so that the obtained number becomes a perfect cube.

$135 \div 5 = 27 = 3^3$ is a perfect cube.

Hence the smallest number by which 135 should be divided to make a perfect cube is 5.

5	135
3	27
3	9
	3

(iv) 192

$192 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times 3$

$= 2^3 \times 2^3 \times 3$

Here, the prime factor 3 is not grouped as a triplet. Hence, we divide 192 by 3, so that the obtained number becomes a perfect cube.

$192 \div 3 = 64 = 4^3$ is a perfect cube

Hence the smallest number by which 192 should be divided to make a perfect cube is 3.

2	192
2	96
2	48
2	24
2	12
2	6
	3

(v) 704

$\underline{704} = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times 11$

$= 2^3 \times 2^3 \times 11$

Here, the prime factor 11 is not grouped as a triplet. Hence, we divide 704 by 11,

so that the obtained number becomes a perfect cube.

Thus, $704 \div 11 = 64 = 4^3$ is a perfect cube

2	704
2	352
2	176
2	88
2	44
2	22
	11

Hence the smallest number by which 704 should be divided to make a perfect cube is 11.

4. Parikshit makes a cuboid of plasticine of sides 5 cm, 2 cm, 5 cm. How many such cuboids will he need to form a cube?

Answer: Given, dimensions of a cuboid are 5 cm, 2 cm, 5 cm

Volume of cuboid = length × breadth × height

= 5 × 2 × 5

= $5^2 \times 2$ cm^3

As there are two 5's and one 2, to make the volume of cuboid as a cube number we need to multiply it by 5 × 2 × 2

Newly formed cube = $5^2 \times 2^1 \times 5 \times 2 \times 2 = 5^3 \times 2^3$ cm^3

Number of cuboids required = $\frac{\text{Volume of cube}}{\text{Volume of cuboid}}$

Number of cuboids required = $\frac{5^3\ x\ 2^3}{5^2\ x\ 2}$ = 5 × 2 × 2 = 20

∴ Number of cuboids required to make a cube = 20

Exercise 7.2

1. Find the cube root of each of the following numbers by prime factorization method.
(i) 64 (ii) 512 (iii) 10648 (iv) 27000 (v) 15625 (vi) 13824 (vii) 110592 (viii) 46656

(ix) 175616 (x) 91125

Answer: To find the cube root of a number, the factors in the prime factorization of the number should be grouped as triplets.

(i) 64

64 = <u>2 × 2 × 2</u> × <u>2 × 2 × 2</u>

= $2^3 \times 2^3$

$\sqrt[3]{64} = 2 \times 2 = 4$

2	64
2	32
2	16
2	8
2	4
2	2
1	1

(ii) 512

2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
1	1

$512 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2}$

$= 2^3 \times 2^3 \times 2^3$

$\sqrt[3]{512} = 2 \times 2 \times 2 = 8$

(iii) 10648

2	10648
2	5324
2	2662
11	1331
11	121
11	11
1	1

$10648 = \underline{2 \times 2 \times 2 \times 11 \times 11 \times 11}$

$= 2^3 \times 11^3$

$\sqrt[3]{10648} = 2 \times 11 = 22$

(iv) 27000

2	27000
2	13500
2	6750
5	3375
5	675
5	135
3	27
3	9
3	3
1	1

$27000 = \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3} \times \underline{5 \times 5 \times 5}$

$= 2^3 \times 3^3 \times 5^3$

$\sqrt[3]{27000} = 2 \times 3 \times 5 = 30$

(v) 15625

5	15625
5	3125
5	625
5	125
5	25
5	5
1	1

$15625 = \underline{5 \times 5 \times 5} \times \underline{5 \times 5 \times 5}$

$= 5^3 \times 5^3$

$\sqrt[3]{15625} = 5 \times 5 = 25$

(vi) 13824

13824 = $\underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3}$

$= 2^3 \times 2^3 \times 2^3 \times 3^3$

$\sqrt[3]{13824} = 2 \times 2 \times 2 \times 3 = 24$

2	13824
2	6912
2	3456
2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
3	3
1	1

(vii) 110592

110592 = $\underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3}$

$= 2^3 \times 2^3 \times 2^3 \times 2^3 \times 3^3$

$\sqrt[3]{110592} = 2 \times 2 \times 2 \times 2 \times 3 = 48$

2	110592
2	55296
2	27648
2	13824
2	6912
2	3456
2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
3	3
1	1

(viii) 46656

$46656 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3} \times \underline{3 \times 3 \times 3}$

$= 2^3 \times 2^3 \times 3^3 \times 3^3$

$\sqrt[3]{46656} = 2 \times 2 \times 3 \times 3 = 36$

2	46656
2	23328
2	11664
2	5832
2	2916
2	1458
3	729
3	243
3	81
3	27
3	9
3	3
1	1

(ix) 175616

$175616 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{7 \times 7 \times 7}$

$= 2^3 \times 2^3 \times 2^3 \times 7^3$

$\sqrt[3]{175616} = 2 \times 2 \times 2 \times 7 = 56$

2	175616
2	87808
2	43904
2	21952
2	10976
2	5488
2	2744
2	1372
2	686
7	343
7	49
7	7
1	1

(x) 91125

$91125 = \underline{5 \times 5 \times 5} \times \underline{3 \times 3 \times 3} \times \underline{3 \times 3 \times 3}$

$= 5^3 \times 3^3 \times 3^3$

$\sqrt[3]{91125} = 5 \times 3 \times 3 = 45$

5	91125
5	18225
5	3645
3	729
3	243
3	81
3	27
3	9
3	3
1	1

2. State true or false
(i) Cube of any odd number is even
(ii) A perfect cube does not end with two zeros
(iii) If square of a number ends with 5, then its cube ends with 25
(iv) There is no perfect cube which ends with 8
(v) The cube of a two digit number may be a three digit number
(vi) The cube of a two digit number may have seven or more digits
(vii) The cube of a single digit number may be a single digit number

Answer:

(i) Cube of any odd number is even.

False

Reasoning: Cubes of odd numbers are odd. Cubes of even numbers are even.

(ii) A perfect cube does not end with two zeros.

True

Reasoning: Perfect cube may end with 3 zeros (or) groups of 3 zeros.

(iii) If square of a number ends with 5, then its cube ends with 25.

False

Reasoning: It is not always necessary that if the square of a number ends with 5, then its cube will end with 25.

For example, the square of 5 is 25, and 25 has its unit digit as 5. The cube of 5 is 125. However, the square of 15 is 225 and also has its unit place digit as 5 but the cube of 15 is 3375 which does not end with 25.

(iv) There is no perfect cube which ends with 8.

False

Reasoning: The cubes of all the numbers having their unit place digit as 2 will end with 8.

For example: The cube of 12 is 1728 and the cube of 22 is 10648.

(v) The cube of a 2-digit number may be a 3-digit number.

False

Reasoning: Cube of a 1-digit number may have 1-digit to 3-digits. Cube of a 2-digit number may have 4-digits to maximum 6-digits.

(vi) The cube of a 2-digit number may have seven or more digits.

False

Reasoning: Cube of a 1-digit number may have 1-digit to 3-digits. Cube of a 2-digit number may have 4-digits to maximum 6-digits.

(vii) The cube of a single-digit number may be a single-digit number.

True

Reasoning: Some examples $1^3 = 1$ and $2^3 = 8$

(viii) Cube of any odd number is even.

False

Reasoning: Cubes of odd numbers are odd. Cubes of even numbers are even.

(ix) Cube of any odd number is even.

False

Reasoning: Cubes of odd numbers are odd. Cubes of even numbers are even.

(x) Cube of any odd number is even.

False

Reasoning: Cubes of odd numbers are odd. Cubes of even numbers are even.

3. You are told that 1,331 is a perfect cube. Can you guess without factorisation what is its cube root? Similarly, guess the cube roots of 4913, 12167, 32768

Answer:

By grouping the digits of the number into triplets starting from one's digit

(i) 1331

Step 1: 1 = Group 2 and 33<u>1</u> = Group 1

Step 2: From group 1, one's digit of the cube root can be identified.

33<u>1</u>= One's digit is 1

Hence cube root one's digit is 1.

Step 3: From group 2, which is 1 only.

Hence cube root's ten's digit is 1.

So, we get $\sqrt[3]{1331} = 11$.

(ii) 4913

Step 1: 4 = Group 2 and 913 = Group 1

Step 2: From group 1, which is 913.

913 = One's digit is 3

We know that 3 comes at the one's place of a number only when its cube root ends in 7. So, we get 7 at the one's place of the cube root. (Refer to table 7.2 INFERENCE)

Step 3: From Group 2, which is 4.

$1^3 < 4 < 2^3$

Taking lower limit. Therefore, the ten's digit of cube root is 1.

So, we get $\sqrt[3]{1331} = 17$

(iii) Similarly, we get $\sqrt[3]{12167} = 23$

(iv) Similarly, we get $\sqrt[3]{32768} = 32$